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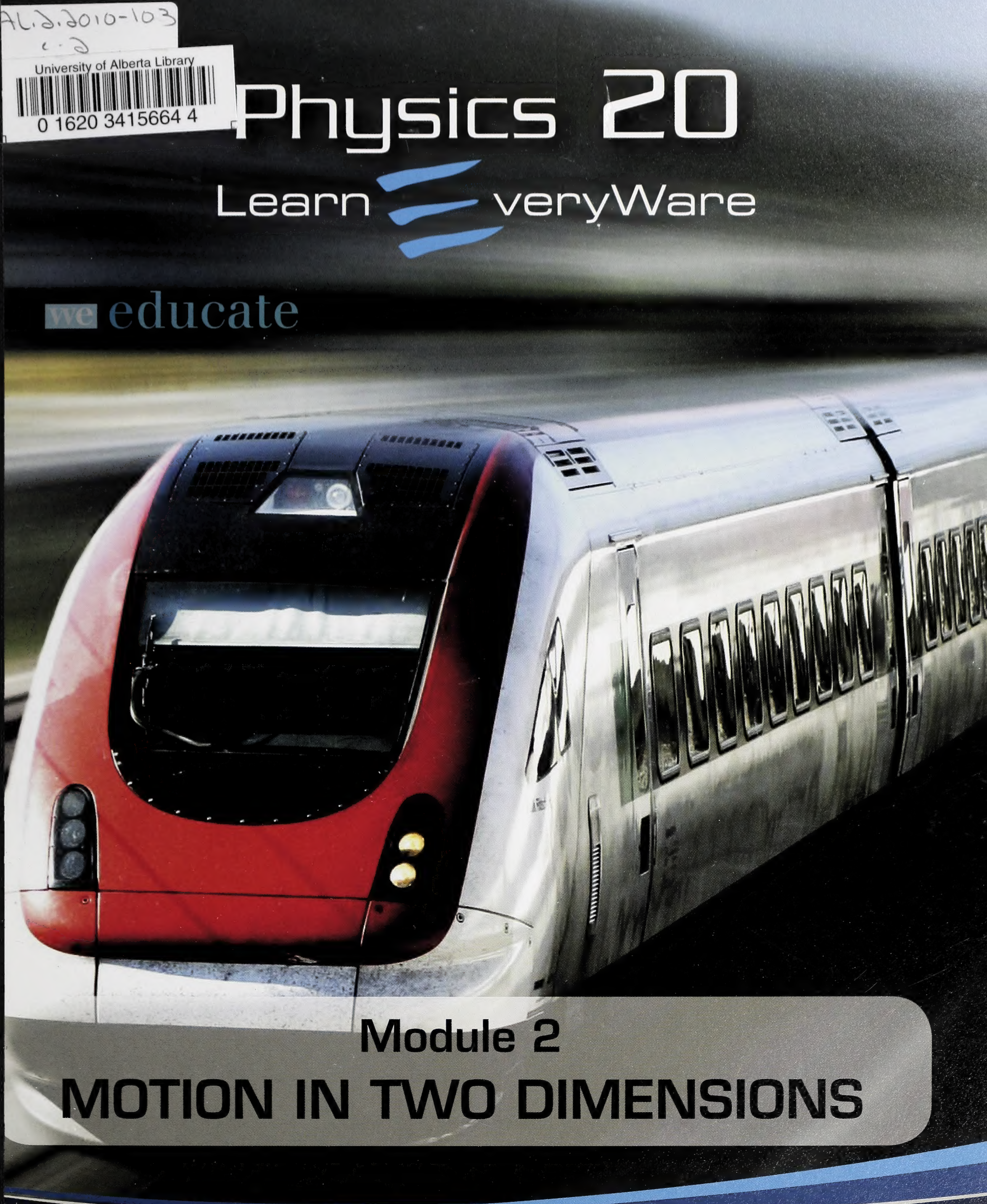

Module 2
MOTION IN TWO DIMENSIONS

Rocky View School Division

Red Deer Catholic Regional Schools

Edmonton Public Schools

Pembina Hills Regional School Division No.7

Black Gold Regional Schools

Peace Wapiti School Division No.76

Calgary Board of Education

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Physics 20

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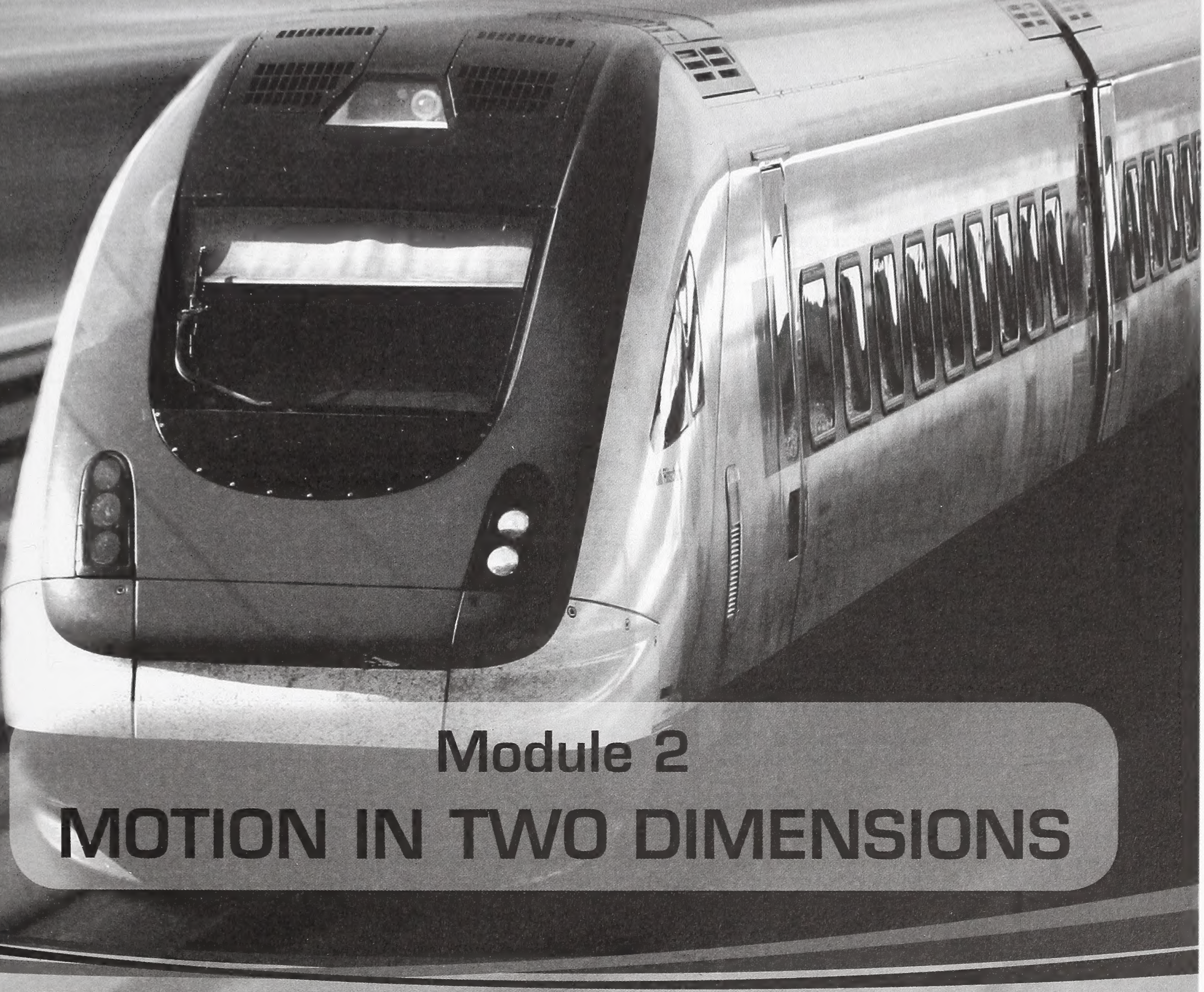

Module 2

MOTION IN TWO DIMENSIONS

Red Deer Catholic Regional Schools

EDMONTON PUBLIC SCHOOLS

Calgary Board of Education

Alberta Education

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- Black Gold Regional Schools
- Calgary Board of Education
- Edmonton School District No. 7
- Peace Wapiti School Division No. 76
- Pembina Hills Regional Division No. 7
- Red Deer Catholic Regional Division
- Rocky View School Division No. 41

Physics 20
Module 2: Motion in Two Dimensions
Student Module Booklet
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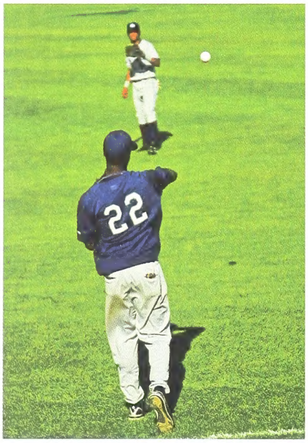
Module 2—Motion in Two Dimensions

Module Introduction



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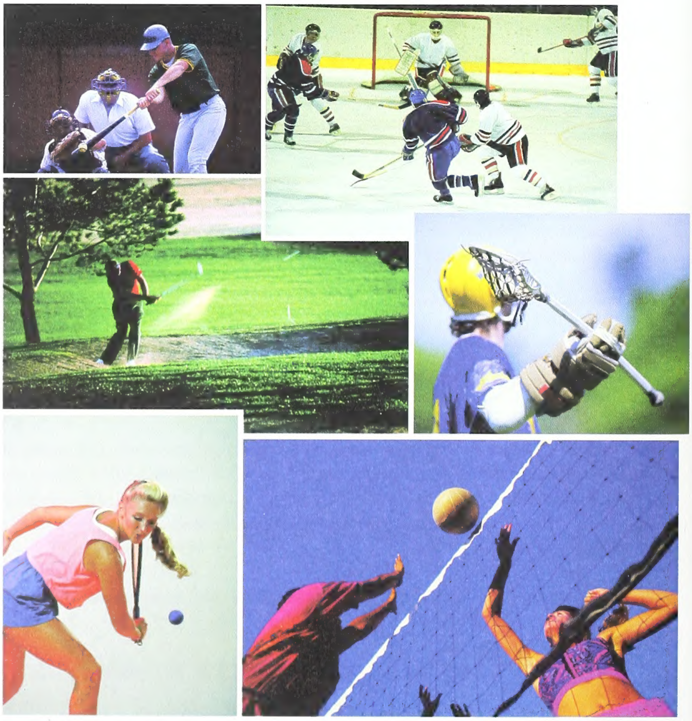
At what angle should you throw a ball to get the maximum distance? What if you want the ball to arrive in the shortest time, such as trying to throw a person out at the plate?

Analyzing situations such as the path of a ball that is thrown where motion occurs in two dimensions is the central theme of this module. You will use the kinematic formulas and the components of vectors you have learned in Module 1 and expand your ability to add vectors and their components graphically. You will develop an ability to solve problems where an object is accelerating in one direction and has constant velocity in another direction. Your confidence in your ability to solve physics questions will increase as you progress through the labs and lessons. The skills you master in Module 2 will be used again and again in your future physics studies.

Big Picture

How many sports can you think of where nothing moves? Now think about sports where something not only moves, but moves along an irregular path. Can you think of more than the sports that are shown in this collage? For some of the moving objects (such as in tennis or hockey), the equipment has changed a lot through the years. Do you think many professional tennis players still use wooden racquets? How about hockey players—are many still using wooden sticks? When a scientific approach is applied to sports equipment, it changes the sport. A scientific approach also affects most areas of our lives. How does physics come into play? Would you like the wings of the airplane you're flying in to fall off? Physics helped the engineers build a plane as strong as it's needed yet not any heavier than needed.



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In this module you will analyze motion along paths that are not straight lines. You will see how the motion of an object can be calculated and how it can be applied in sports and in many other real-life situations. As you are working in Module 2, you should keep the following questions in mind. They should help you fit the world of kinematics into your everyday world.

Essential questions for you to consider in this module are as follows:

- How do vectors help explain two-dimensional motion?
- In what ways can data be used to distinguish differing types of motion?
- In what ways can mathematics be used to help understand motion?
- How can observations and questions help explain what is seen in everyday life?

In This Module

Lesson 1—Graphical Vector Addition in Two Dimensions

In this lesson you will learn how to graphically add two vectors in two dimensions.

You will explore the following essential question in this lesson and the accompanying lab:

- How do vectors help explain two-dimensional motion?

Lesson 2—Vector Components and Cartesian and Navigational Vector Directions

In this lesson you will learn to calculate the magnitude and direction of the sum of two vectors, in two dimensions, when given the magnitudes and direction of the two vectors to be added.

You will explore the following essential questions:

- How do you mathematically determine the magnitude and direction of the sum of two vectors, such as air velocity and wind velocity, for a flying plane?
- How do you calculate the components of the two vectors to be added when given the magnitudes and directions?

Lesson 3—Understanding and Predicting the Motion of a Projectile

In this lesson you will explore how independent vertical and horizontal motions can be analyzed with kinematics equations to predict the two-dimensional position and velocity of a projectile at various times.

You will explore the following essential question:

- How do you use kinematic equations to determine the range (horizontal displacement) and the time of flight for a projectile with constant acceleration due to gravity?

Lesson 4—Solving Projectile Problems

In this lesson you will learn that in the absence of air resistance, projectile motion of an object can be described as a superposition of two motions—a horizontal motion with constant velocity and a vertical motion with uniform acceleration due to gravity. You will also learn to apply the relevant kinematics equation of motion with constant acceleration or uniform motion to solve projectile problems.

You will explore the following essential question.

- How can independent vertical and horizontal motions be analyzed with kinematic equations to predict the two-dimensional position and velocity of a projectile at various times?

Module 2 Assessment

The assessment for Module 2 consists of four (4) assignments, as well as a final module report.

- Module 2: Lesson 1 Assignment
- Module 2: Lesson 2 Assignment
- Module 2: Lesson 3 Assignment
- Module 2: Lesson 4 Assignment
- Module 2 Project

Module 2 Project: Technology in Sports

In Module 2 you will be required to complete a module project that relates how technology has changed sports. Your project can take the form of a speech, letter, report, or multimedia presentation that discusses your findings. You will be asked to complete portions of the project in each lesson.

Your final presentation will be graded according to these guidelines.

Scoring Rubric for Examples

1. **Poor:** The presentation is irrelevant or so scant, overgeneralized, or inaccurate that it indicates a poor or almost complete lack of understanding. It contains major errors or is largely inappropriate.

2. **Limited:** The presentation is unfocused or inappropriate, potentially relevant but contains inaccuracies or extraneous detail. It shows an understanding that is confused or oversimplified.

3. **Satisfactory:** The presentation is largely relevant but may contain some minor factual errors, or there may be a mixture of relevant and extraneous information. It shows a general understanding of dynamics as it relates to sports.

4. **Proficient:** The presentation is purposeful, usually specific, relevant, and accurate, indicating a solid understanding of how dynamics affects sports.

5. **Excellent:** The presentation is comprehensive, specific, relevant, and accurate, and it demonstrates a superior understanding of how dynamics affects sports.

Each of the preliminary parts of the project will be graded on a 2-point scale: 0 points if the part shows little or no effort expended, 1 point for some effort, and 2 points if considerable effort is shown.

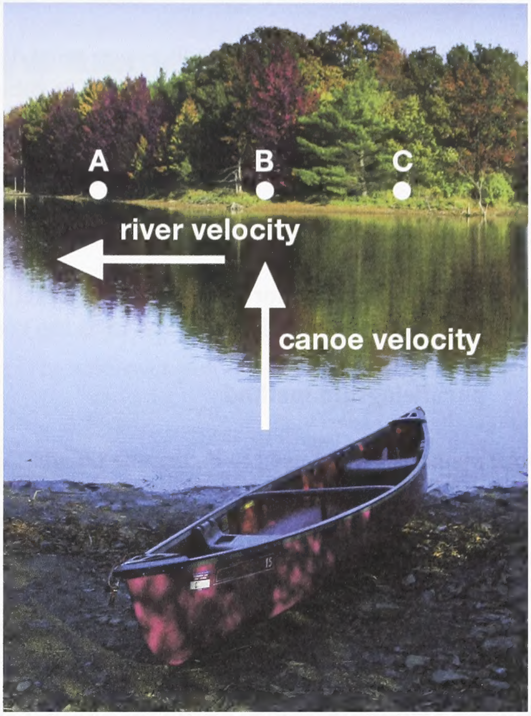
Lesson 1—Graphical Vector Addition in Two Dimensions

Get Focused

While on a camping trip, you decide to explore the beach on the other side of a nearby river. You launch a canoe with a velocity directed straight across the river, toward point B on the diagram to the left. Given the velocity of the river, where do you think the canoe will land on the other side—point A, B, or C?

If you said point A, you understand that both the velocity of the canoe and the velocity of the river contribute to the observed velocity of the boat. Given that information, how would you point the boat so that it would end up at point B?

In this lesson you will learn how to use graphical vector addition to answer such questions.



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Specifically, you will explore the following essential question in this lesson:

- How do vectors help explain two-dimensional motion?

Module 2: Lesson 1 Assignments

In this lesson you will complete the Lesson 1 Assignment in the Module 2 Assignment Booklet.

- Lab—LAB 2 and LAB 3
- Try This—TR 1
- Reflect and Connect
- Discuss

The other questions in this lesson are not marked by the teacher; however, you should still answer these questions. The Self-Check and Try This questions are placed in this lesson to help you review important information and build key concepts that may be applied in future lessons.

After a discussion with your teacher, you must decide what to do with the questions that are not part of your assignment. For example, you may decide to submit to your teacher the responses to Try This questions that are not marked. You should record the answers to all the questions in this lesson and place those answers in your course folder.

Explore

There are two methods for graphically constructing the sum of two vectors, which is referred to as the **resultant**:

- **tip-to-tail method**
- **parallelogram method**

Both methods can be used to determine the velocity of the canoe and the velocity of the water that was used in the previous example. The resultant in the case of the canoe crossing the river would be the observed velocity of the boat. Both methods can be used to add more than two vectors by first adding any two vectors and then adding their resultant to a third vector using the same method.

resultant: a vector sum of two or more other vectors

parallelogram method: a method of adding vectors, where the two vectors' tails start at the same point and define adjacent sides of a parallelogram

The diagonal of the parallelogram that starts where the tails meet is the resultant.

tip-to-tail method: a method of adding vectors where the second vector begins at the end (tip) of the first vector

The resultant is the vector from the tail of the first vector to the tip of the second.

Adding Two-dimensional Vectors—Tip-to-Tail Method

Lesson 1 Lab: Graphical Vector Addition

Introduction

This simulation lets you see the addition of vectors using the tip-to-tail method and the parallelogram method in turn.

You can learn more about the simulation and how to use it by reading the Show Me found at the top of the simulation screen.

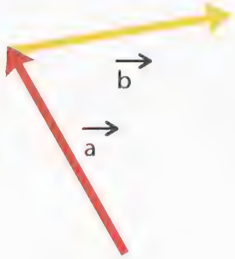
Method 1: Tip to Tail

Recall the Focus

How can two vectors be added graphically using the tip-to-tail method of vector addition?

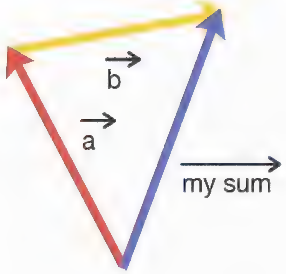
Go to **www.learnalberta.ca**. You may be required to input a username and password. Contact your teacher for this information. Enter the search terms "vector addition" into the search bar. Choose the item called "Vector Addition: Graphical"; then continue with the procedure.

Procedure



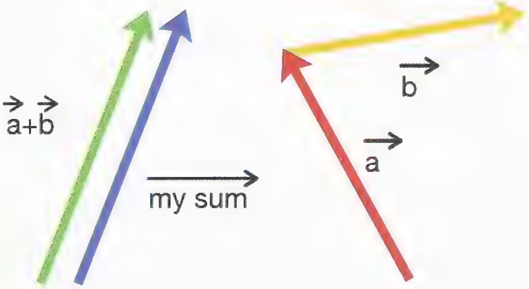
The simulation will be used to demonstrate the tip-to-tail method of vector addition.

- If the simulation screen is not empty, clear it by pressing the "Reset" button ( ).
- Draw two vectors in the simulation window using the "Vector" button (vector). To draw the second vector, click "Vector" again. The simulation will label the two vectors $\vec{a}$ and $\vec{b}$.
- Arrange the two vectors so that the tail end of vector $\vec{b}$ is aligned with the tip of vector $\vec{a}$ as shown here. Clicking on the middle of the vector enables you to drag it to wherever you want. Clicking on the very end of a vector allows you to reposition that end.
- To draw the resultant, select the "Vector Sum" button (Vector Sum) and draw the vector from the free tail end of the first vector to the tip of the arrow on the second vector. The simulation draws your resultant in blue and labels it "my sum."



There is an easy way to remember in which direction to draw the resultant arrow. Think of the two vectors as one displacement following another. The resultant is the overall (net) displacement, from the point where the first displacement starts to the point where the second displacement finishes.

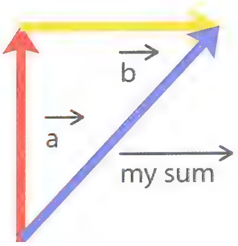
- Once you have drawn your resultant, the "Answer" button (Answer) becomes active. When you press this button, the simulation will draw the correct resultant in green and label it $\vec{a}+\vec{b}$. Compare your resultant to the correct resultant by dragging yours next to the correct one. Can you make the two overlap completely? You can also move the correct resultant.



Observations and Analysis

LAB 1. Using the simulation, create the following $\vec{a}$ and $\vec{b}$ vectors. Then identify which resultants are correct and which are incorrect.

a.



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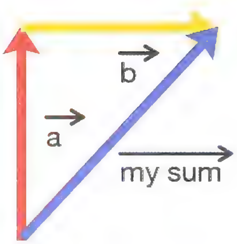

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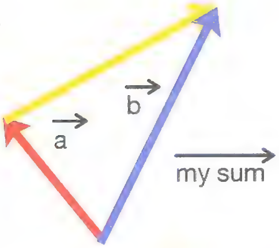
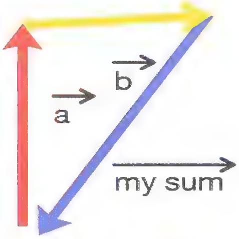
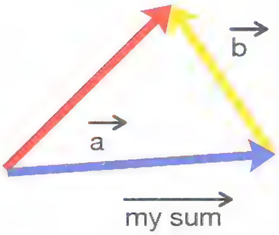


d.



e.



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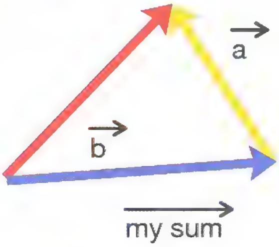
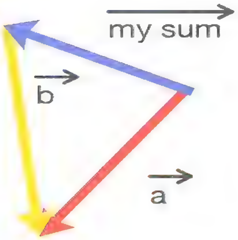


g.



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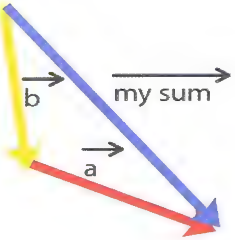
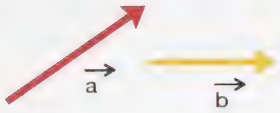


Module 2: Lesson 1 Assignment

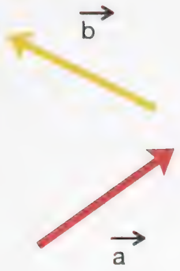
Remember to submit the answer to LAB 2 to your teacher as part of your Lesson 1 Assignment in the Module 2 Assignment Booklet.

LAB 2. Sketch by hand the tip-to-tail method of vector addition and the resultant vector for the following vectors. Use the "Vector Addition: Graphical" simulation to verify your answers.

a.



b.



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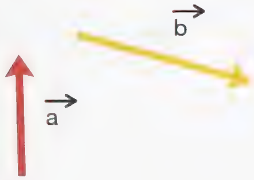
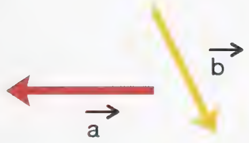


d.



Method 1 Summary

How can two vectors be added graphically using the tip-to-tail method of vector addition?

Vectors are added by placing the tail of the second vector at the tip of the first vector. The sum (resultant) is the vector that shares the first vector's tail and the second vector's tip.

Read

In paddling a canoe across a river, as shown in the example in the Get Focused, both the river and the canoe had velocities. How do you get the value of the resultant velocity? Read "Adding Two-dimensional Vectors Graphically" on pages 80 and 81 in your textbook to see the steps used in the tip-to-tail method to find the numerical value of the resultant of two vectors.

To see how this works in a practical problem of air speed in a plane flight so that you can do it yourself, go to your Physics 20 Multimedia DVD, and open the "Adding Vectors Graphically" simulation. You will need to use a ruler, a pencil, an eraser, a protractor, and a sheet of lined paper as you follow along with the simulation.

Module 2: Lesson 1 Assignment

Remember to submit the answer to TR 1 to your teacher as part of your Lesson 1 Assignment in the Module 2 Assignment Booklet.

Try This

TR 1. Using the tip-to-tail method of vector addition, complete "2.2 Check and Reflect" question 8 on page 90 of your textbook.

Read

What happens if you have to add more than two vectors? Read the last third of page 81 and the first half of page 82 in your textbook to see an example of how the tip-to-tail method is used to find the numerical value of the resultant of more than two vectors.

Self-Check

SC 1. Complete practice problem 2 on page 82 of your textbook.

Check your work with the answer in the appendix.

Method 2: Parallelogram

Some people prefer to use the parallelogram method instead of the tip-to-tail method. Either method works. Although the textbook uses the tip-to-tail method, see what you think of the parallelogram method.

Re-open the "Vector Addition: Graphical" simulation at **www.learnalberta.ca**; then continue with the procedure.

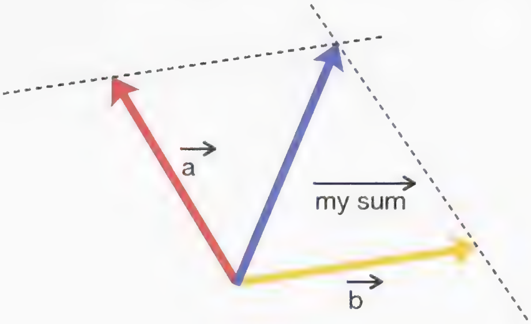
Procedure

The simulation will be used to demonstrate the parallelogram method of vector addition.

- If the screen is not empty, clear it by pressing the "Reset" button (⟳).
- Draw two vectors in the window. The simulation will label the two vectors $\vec{a}$ and $\vec{b}$.

- Join the two vectors tail end to tail end. Think of these two vectors as spanning a parallelogram—for example, forming two sides of a parallelogram whose other two sides still need to be constructed.

- The simulation allows you to draw the remaining two sides of the parallelogram with dotted lines. The resultant vector goes from the point where the tail ends of the vectors join to the opposite corner of the parallelogram. To make sure that the dotted lines are parallel to the given vectors, draw the lines initially on top of the two vectors. Then move them into their correct positions by making them intersect the tips of the respective vectors.



- Finally, you can compare your resultant to the correct resultant drawn by clicking on the "Answer" button (Answer) in the simulation as described at the end of the section on the tip-to-tail method.

Observations and Analysis

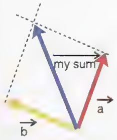
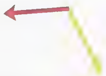
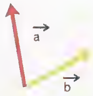
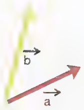
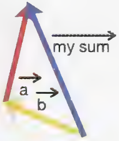
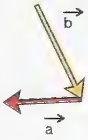
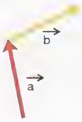
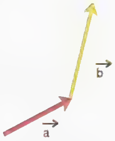
Module 2: Lesson 1 Assignment

Remember to submit the answer to LAB 3 to your teacher as part of your Lesson 1 Assignment in the Module 2 Assignment Booklet.

LAB 3. Complete the following steps.

- **Step 1:** Draw the parallelogram for the vectors in the top row of the table.
- **Step 2:** Draw the resultant for the vectors in the top row of the table.
- **Step 3:** Draw the resultant for the vectors in the bottom row of the table using the tip-to-tail method.

Examples of the parallelogram method and tip-to-tail method have been provided.

Parallelogram Method	Example:  	a. 	b.  	c. 
Tip-to-Tail Method	Example:  	d.  	e.  	f. 

Method 2 Summary

How can vectors be added using the parallelogram method of vector addition?

Vectors are added by placing the tail of the two vectors together and completing the parallelogram these two vectors define. The sum (resultant) is the vector that shares these vectors' tails and the opposite vertex of the parallelogram.

Self-Check

SC 2. Refer to LAB 3 in order to answer the following questions.

a. What do you notice about the magnitude and direction of each vector $\vec{a}$ in the bottom (tip-to-tail method) row of Lab 3 compared to the vector $\vec{a}$ in the table just above it?

b. What do you notice about the magnitude and direction of each vector $\vec{b}$ in the bottom (tip-to-tail method) row of Lab 3 compared to the vector $\vec{b}$ in the table just above it?

c. What do you notice about the magnitude and direction of each resultant vector in the bottom (tip-to-tail method) row of Lab 3 compared to the resultant vector in the table just above it?

d. Are the magnitude and direction of the resultant vector different if you use the tip-to-tail method instead of the parallelogram method?

e. Which method do you prefer to use? Explain why.

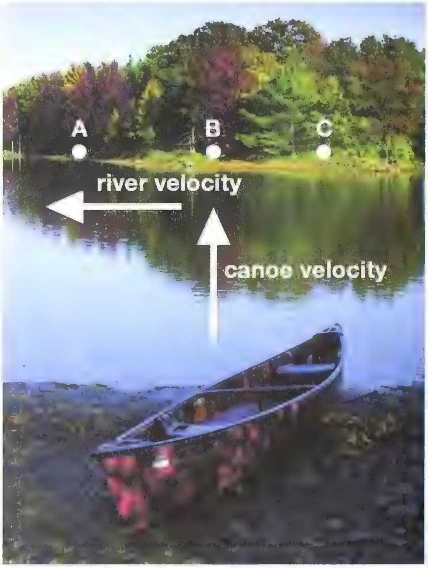
Check your work with the answer in the appendix.

Module 2: Lesson 1 Assignment

Remember to submit the answers to Reflect and Connect to your teacher as part of your Lesson 1 Assignment in the Module 2 Assignment Booklet.

Reflect and Connect

Recall the canoe problem from the beginning of the lesson. Using the tip-to-tail method of vector addition, construct a vector diagram that shows how the canoe would need to be directed in order to get to points A, B, and C. Consider how each of the three vector diagrams (triangles) demonstrates the tip-to-tail method of vector addition.



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Module 2: Lesson 1 Assignment

Remember to submit the answers to Discuss to your teacher as part of your Lesson 1 Assignment in the Module 2 Assignment Booklet.

Discuss

Using the two vectors $\vec{a}$ and $\vec{b}$, you should be able to show that $\vec{a}+\vec{b}$ and $\vec{b}+\vec{a}$ give the same resultant. Illustrate both scenarios using a vector diagram. Post your diagram, and explain your proof in the discussion area.

Reflect on the Big Picture

In this lesson you learned two ways to put vectors together. You worked with the geometry of vectors, but in the context of describing how objects move. Did you find that actually drawing the vectors by hand helped you understand their addition?

To help you reflect on your learning experience in this lesson, complete at least one of the following three activities:

- What do you think you learned in this lesson? Look over the lesson, and fill in a learning log like the one that follows.

Learning Log
Describe things that you know now that you didn't know before.
Describe things that you still need to work on.
Describe ways to increase your skills and knowledge.

- Draw or paint a picture or create a sculpture that contains the essence of adding two vectors. Add a short explanation to your course folder of how your artwork explains vector addition.

- Write a short paragraph explaining to your friends the way to paddle a canoe to go straight across a rapidly flowing stream. Incorporate the concept of vector addition as subtly as possible.

Store your completed reflection in your Physics 20 course folder.

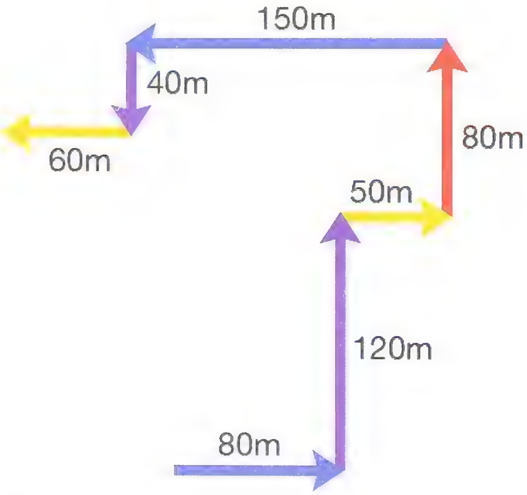
Going Beyond

GPS tracking can be used to monitor the motion and location of a taxi to improve response times.

© 2008 Jupiterimages Corporation

Try This

TR 2. The following map shows the motion of a taxi as it moves through a city. Show how you could add all of these vectors using a single triangle and the tip-to-tail method. Determine the magnitude and polar positive direction of the taxi's displacement (the resultant).



Module 2: Lesson 1 Assignment

Make sure you have completed all of the questions for the Lesson 1 Assignment. Check with your teacher about whether you should submit your assignment now or wait until all of the Module 2 assignments have been completed.

Choose four sporting activities that require equipment. Do some preliminary research about the changes in the equipment used in each sport in the past 25 or so years. Store your findings in your Physics 20 course folder. You will submit this research to your teacher at the end of Lesson 4 as part of your Module 2 Project.

Lesson Summary

You explored the following essential question in this lesson:
- How do vectors help explain two-dimensional motion?

There are two methods for graphically constructing the sum of two or more vectors:
- tip-to-tail method
- parallelogram method

Both methods will produce the "sum of two or more vectors," which is referred to as the resultant. The resultant vector illustrates the magnitude and direction of the sum of two or more vectors.

Lesson Glossary

parallelogram method: a method of adding vectors, where the two vectors' tails start at the same point and define adjacent sides of a parallelogram

The diagonal of the parallelogram that starts where the tails meet is the resultant.

resultant: a vector sum of two or more other vectors

tip-to-tail method: a method of adding vectors, where the second vector begins at the end (tip) of the first vector

The resultant is the vector from the tail of the first vector to the tip of the second.

Lesson 2—Vector Components and Cartesian and Navigational Vector Directions

Get Focused

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While booking a round-trip air ticket to Vancouver to go skiing at Grouse Mountain, you notice that it will take 1 h and 28 min to get from Calgary to Vancouver and that it will only take 1 h and 20 min to fly back. Why do you think it takes 8 min more to get there than it does to get back? The distance between Calgary and Vancouver doesn't change. You are flying on the same aircraft at exactly the same speed (approximately 750 km/h, relative to the air) both ways. What could be going on?

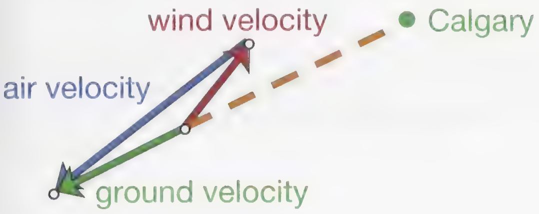
On both trips, your air velocity and distance of travel are identical. The time is different, however, because of the wind velocity. When you consider the air velocity (how fast the plane moves in the air) and the wind velocity together, you are able to show that the plane has a greater "ground velocity" when it flies toward the east. Here are the **vector diagrams** demonstrating that the magnitude of the ground velocity is greater when the plane travels from Vancouver to Calgary—even if the wind velocity and air velocity have a constant magnitude.

vector diagram: a diagram using vectors to show motion

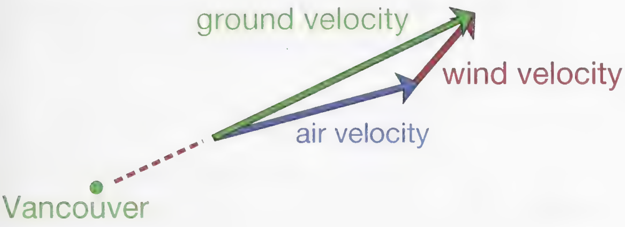
Vector diagrams may instead be used to show forces, fields, or momentum.

© jamie cross/BigStockPhoto

Travelling from Calgary to Vancouver:



Travelling from Vancouver to Calgary (8 min faster):



In aviation, wind velocity describes how fast the wind is moving relative to the ground. In the two previous cases, the wind velocity is constant. Air velocity, on the other hand, is a measure of how fast the plane moves relative to the air (controlled by the jet engines). And finally, ground velocity describes how fast the plane moves relative to the ground (a combination of air velocity and wind velocity).

In this lesson you will explore the following essential questions:

- How do you mathematically determine the magnitude and direction of the sum of two vectors, such as air velocity and wind velocity, for a flying plane?

- How do you calculate the components of the two vectors to be added when given the magnitudes and directions?

Module 2: Lesson 2 Assignments

Your Lesson 2 Assignment in the Module 2 Assignment Booklet requires you to submit a response to the following:

- Try This—TR 1, TR 2, TR 3, TR 4, TR 5, and TR 6
- Discuss

You must decide what to do with the questions that are not marked by the teacher.

Remember that these questions provide you with the practice and feedback that you need to successfully complete this course. You should respond to all the questions and place those answers in your course folder.

Explore

This lesson began with questions posed in Get Focused. Lesson 2 Lab: Numerical Vector Addition includes a simulation to numerically calculate the answers to those questions. You will be required in the lab to use vector concepts introduced in Lesson 1 of Module 1. Read your textbook and answer the following Self-Check question to review those concepts.

Read

Review “Motion in Two Dimensions” on pages 76 to 78 of your textbook. The Cartesian method is capitalized because it is named after a person, René Descartes, an early French philosopher and mathematician.

Self-Check

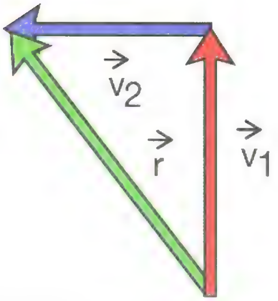
SC 1. Use the diagram below to answer the following questions:

a. Which vector is the x component?

b. Which vector is the y component?

c. What is the direction of the green $\vec{r}$ vector in the Cartesian method?

d. If the x direction was east and the y direction was north, what is the direction of the green $\vec{r}$ vector measured from west in the navigator method?



Check your work with the answer in the appendix.

Lesson 2 Lab: Numerical Vector Addition

Introduction

In the example of the commercial aircraft flight between Calgary and Vancouver in Get Focused, there are two different velocities that contribute to the ground velocity:

- air velocity, which is the plane's velocity relative to the air
- wind velocity, which is the wind's velocity relative to the ground

Adding these two vectors is not as straightforward as it may seem. If the air velocity is 10.0 m/s and the wind velocity is 5.0 m/s, and they are not moving in the same direction, the plane's ground velocity will not be 10.0 ms + 5.0 m/s = 15.0 m/s. You must add the two velocities as vector quantities with both a magnitude and direction.

The simulation used for this lab adds two or more vectors. It will help you to calculate the magnitude and direction of the sum of two vectors. You can learn more about the simulation and how to use it by reading the Show Me found at the top of the simulation screen.

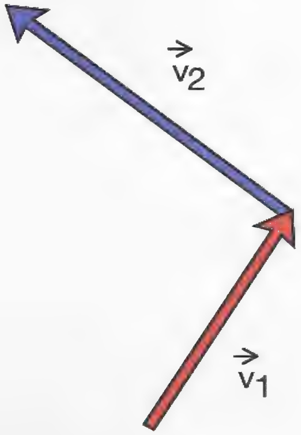
Problem

How do you determine and illustrate the sum of two vectors?

Go to **www.learnalberta.ca**. You may be required to input a username and password. Contact your teacher for this information. Enter the search terms "vector addition" into the search bar. Choose the item called "Vector Addition: Numerical" from the list. Open thesimulation and adjust the margins, if needed, so the black areas are minimized.

Procedure

The simulation will be used to determine a plane's velocity relative to the ground when both the wind velocity and the air velocity are known. For example, how would you determine the ground velocity of a plane that is flying with an air velocity of 130 m/s at 140° in the polar direction while the wind velocity is 100 m/s at 60°?

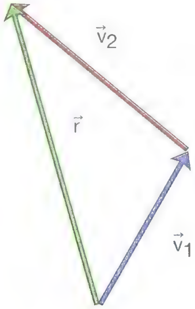


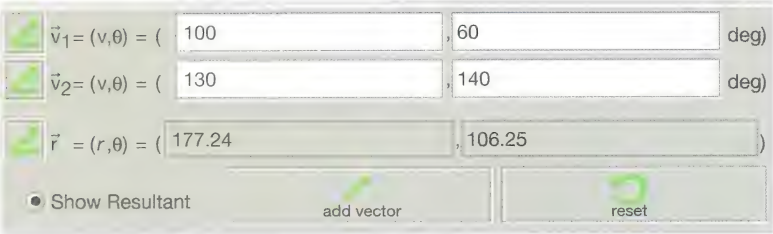
- If the window shows more than two vectors, clear it by pressing the “Reset” button (⟳).

- Enter the first vector, $\vec{v}_1$, with a magnitude of 100 m/s at an angle of 60° in the polar (positive) mode 🟩).

- Enter the second vector, $\vec{v}_2$, with a magnitude of 130 m/s at an angle of 140° in the polar (positive) mode (🟩), and drag both vectors to the centre of the screen.

- After entering the two vectors, click the "Show Resultant" button labelled to display the resultant (sum of the vectors). If you use the polar (positive) mode (🟩) for the resultant, you should see a display similar to that below.




$\vec{v}_1$ = (v,θ) = (100 , 60 deg)
$\vec{v}_2$ = (v,θ) = (130 , 140 deg)
$\vec{r}$ = (r,θ) = (177.24 , 106.25)
● Show Resultant

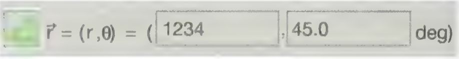
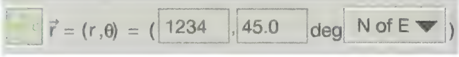
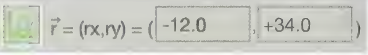
According to the simulation, the plane's velocity relative to the ground would be 177.24 m/s at 106.25°.

Observation and Analysis

LAB 1. Choose the correct answer below. The graphical construction of the resultant illustrates the

A. tip-to-tail method of vector addition
B. parallelogram method of vector addition

LAB 2. The resultant may be displayed in three modes. Click the "Mode Toggle" button (🟩) for the resultant, $\vec{r}$, three times to cycle to the Cartesian mode (🟩). In this mode the resultant's x and y components (r_x, r_y) are displayed as shown below.

Button	Mode	Example
🟩	magnitude and polar positive direction (r, θ)	 $\vec{r}$ = (r,θ) = (1234 , 45.0 deg)
🟩	magnitude and navigational direction (r, θ N of E)	 $\vec{r}$ = (r,θ) = (1234 , 45.0 deg N of E ▼)
🟩	Cartesian components (r_x, r_y)	 $\vec{r}$ = (rx,ry) = (-12.0 , +34.0)

Using the “Mode Toggle” button, describe the plane’s velocity relative to the ground using all three modes.

a. The magnitude and polar (positive) direction of the resultant ($\vec{r}$) is (________ , ________ deg).

b. The magnitude and navigational direction of the resultant ($\vec{r}$) is (________ , ________ E of N).

c. The Cartesian components of the resultant ($\vec{r}$) are (________ , ________).

How is the resultant being calculated? One common method is to analyze and add the vector components.

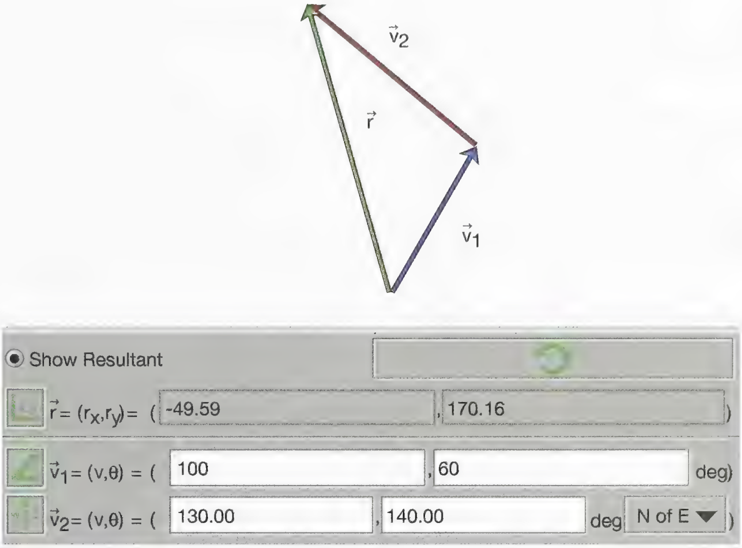
Calculating a Resultant Vector Using Vector Components

You will now calculate the sum of the same two vectors, $\vec{v}_1$ and $\vec{v}_2$, using component analysis.

For the following calculations, you will need to know the components of a vector.

It is particularly easy if the vectors are already given in terms of their x and y components, $(vx, vy)_1$ and $(v_x, v_y)_2$.

However, what if the vectors are given in terms of magnitude and direction $[(v_1, \theta_1)$ and $(v_2, \theta_2)]$?



● Show Resultant

$\vec{r} = (r_x, r_y) =$ (-49.59 , 170.16)

$\vec{v}_1 = (v, \theta) =$ (100 , 60 deg)

$\vec{v}_2 = (v, \theta) =$ (130.00 , 140.00 deg N of E ▾)

Angles are measured in the polar (positive) specification (or navigation N of E).

The magnitude and direction of $\vec{v}_1$ and $\vec{v}_2$ are

- $v_1 = 100$ m/s, $\theta_1 = 60°$
- $v_2 = 130$ m/s, $\theta_2 = 140°$

Go to your Physics 20 Multimedia DVD, and open the "Component Vector Addition" simulation. It will show you in detail how to calculate the resultant. When you have read a screen, click the “Next” arrow to go on. On screens 7 to 9, when the symbols show v_{fx} and v_f, they really mean v_{1x} and v_1.

Read

Read "Determining Components" on pages 83 and 84 of your textbook to see in greater detail another explanation of this skill you saw in the "Component Vector Addition" simulation.

Self-Check

SC 2. Complete practice problem 2 for "Example 2.4" on page 84 of the textbook.

Check your work with the answer in the appendix.

Read

Read "Adding Vectors Using Components" on pages 85 through 89 of your textbook to see an explanation of this skill you saw in the Component Vector Addition simulation.

Lesson 2 Lab: Adding Vectors Graphically

In the previous lesson you added two vectors graphically to find the ground speed in an airplane flight. Any slight variation in measuring lines or angles could give an erroneous answer. Now it is time to calculate the answer mathematically to see how close you were using a guided simulation. You will need a calculator, a pencil, an eraser, and a separate sheet of lined paper. Go to the Physics 20 Multimedia DVD, and open the "Numerical Vector Addition" simulation, and follow the instructions.

Look in your Physics 20 course folder to see your previous work on the air and ground speed problem. How close to the calculated answer was your graphical solution? What are the unique advantages you see to each method?

Now you have a chance to do a vector addition question on your own with the following Self-Check question.

Self-Check

SC 3. Complete practice problem 2 for "Example 2.5" on page 88 of the textbook.

Check your work with the answer in the appendix.

Lab Summary

How do you determine and illustrate the sum of two vectors?

The sum of two vectors is determined using the following steps:

- Determine the components of each vector using the appropriate trigonometric functions.
- Add the respective components to determine the total *x* and total *y* vectors.
- Construct a right triangle using the total *x* and *y* vectors.
- Determine the magnitude of the sum (hypotenuse) using the Pythagorean theorem.
- Determine the direction of the sum using the appropriate trigonometric function.

Self-Check

SC 4. Using the component method, calculate the resultant (sum) of the following two vectors.

$\vec{v}_1$ = 185 m/s, 45° polar (positive)
$\vec{v}_2$ = 95 m/s, 320° polar (positive)

Show all required calculations and diagrams, and identify the direction using the polar (positive) specification. You may use the "Vector Addition: Numerical" simulation at **www.learnalberta.ca** to add the vectors in order to verify the resultant magnitude and direction.

a. Make a vector diagram for $\vec{v}_1$, and calculate the components.

b. Make a vector diagram for $\vec{v}_2$, and calculate the components.

c. Add the components of the resultant vector.

d. Draw the resultant vector.

e. Calculate the resultant magnitude using the Pythagorean theorem.

f. Calculate the resultant direction using the tangent function. Express the direction in terms of the polar (positive) specification.

Check your work with the answer in the appendix.

Module 2: Lesson 2 Assignment

Remember to submit the answers to TR 1 and TR 2 to your teacher as part of your Lesson 2 Assignment in the Module 2 Assignment Booklet.

Try This

TR 1. Using the component method, calculate the resultant (sum) of the following two vectors.

- $\vec{v}_1$ = 175 m/s, 70° polar (positive)
- $\vec{v}_2$ = 200 m/s, 200° polar (positive)

Show all required calculations and diagrams, and identify the direction using the polar (positive) specification. You may use the "Vector Addition: Numerical" simulation at **www.learnalberta.ca** to add the vectors in order to verify the resultant magnitude and direction.

a. Make a vector diagram for $\vec{v}_1$, and calculate the components.

b. Make a vector diagram for $\vec{v}_2$, and calculate the components.

c. Add the components of the resultant vector.

d. Draw the resultant vector using the components.

e. Calculate the resultant magnitude using the Pythagorean theorem.

f. Calculate the resultant direction using the tangent function. Express the direction in terms of the polar (positive) specification.

TR 2. Using the component method, calculate the resultant (sum) of the following two vectors.

- $\vec{v}_1$ = (+135 m/s, –120 m/s) components
- $\vec{v}_2$ = (–200 m/s, –45 m/s) components

Show all required calculations and diagrams, and identify the direction using the polar (positive) specification. You may use the "Vector Addition: Numerical" simulation to add the vectors in order to verify the resultant magnitude and direction.

a. Add the components of the resultant vector.

b. Draw the resultant vector.

c. Calculate the resultant magnitude using the Pythagorean theorem.

d. Calculate the resultant direction using the tangent function. Express the direction in terms of the polar (positive) specification.

Read

Read "Relative Motion" on pages 91 to 100 of your textbook.

Self-Check

SC 5. A jetliner flying at 725 km/h [W] encounters a headwind of 70 km/h [E]. What will be the velocity of the jet relative to the ground?

Check your work with the answer in the appendix.

Lesson 2 Lab: Table Tennis in the Wind

Complete the procedure of "2-3 QuickLab: Table Tennis in the Wind" on page 92 of the textbook.

LAB 3. Answer questions 1, 2, 3, and 4 of "2-3 QuickLab: Table Tennis in the Wind" on page 92 of the textbook.

Read

What happens when the wind blows at an angle to the airplane? This is where the use of components, which you have been practising, becomes a key skill in solving problems. Read "Non-collinear Relative Motion" on pages 93 to 97 of your textbook to see how they are used.

Self-Check

SC 6. An airplane with airspeed of 400 km/h is flying south, but an east wind of 86.0 km/h blows the plane off course.

a. What was the actual velocity of the plane?
b. In what direction should the pilot fly to head due south?

Check your work with the answer in the appendix.

Module 2: Lesson 2 Assignment

Remember to submit the answer to TR 3 to your teacher as part of your Lesson 2 Assignment.

Try This

TR 3. An aircraft has a cruising speed of 100 m/s. On this particular day, a wind is blowing from the west at 75.0 m/s.

a. If the plane were to fly due north, what would be the velocity relative to the ground?

b. If the pilot wishes to have a resultant direction of due north, in what direction should the plane be pointed? What will be the plane's displacement in 1.25 h?

Read

What happens when the motion is on water and the currents are not perpendicular to the direction of travel? Again, components become the key in solving these problems. Read "Relative Motion in the Water" on pages 98 to 100 of your textbook to see how they are used.

Module 2: Lesson 2 Assignment

Remember to submit the answers to TR 4, TR 5, and TR 6 to your teacher as part of your Lesson 2 Assignment in the Module 2 Assignment Booklet.

Try This

TR 4. A dog wishes to swim across a slow-moving stream. The dog can swim at 2.0 m/s in calm water. The current velocity is 3.0 m/s. The distance directly across the stream is 50 m.

a. If the dog points himself directly across the stream, how long will it take to get across the stream?

b. How far downstream will the current have carried the dog when the dog gets to the other side?

c. What was the dog's velocity relative to the bank from where the dog started?

TR 5. For each of the following vectors, calculate the horizontal and vertical components.

a. 25.0 m/s [35.0°] (polar positive)
b. 16.0 m [120°] (polar positive)
c. 125 km/h at 220° (polar positive)

TR 6. A man walked 440 m [50.0°] (polar positive) and then 580 m [185°] (polar positive). The entire trip took 150.0 min.

a. What was the total distance travelled?
b. What was the displacement of the man?
c. What was the average speed of the man (in m/min)?
d. What was the average velocity of the man (in m/min)?

Reflect and Connect

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Before a plane takes off from a Canadian airport, the pilot must file a flight plan with NAV CANADA (a private, non-profit corporation that provides air navigation services). The flight plan should include travel distances, estimated departure and arrival times, weather, stop locations, airport approach plans, and more. The amount of fuel required for the flight must also be indicated on the plan.

Based on what you have learned about vector addition and relative motion in the air, what does a pilot need to consider when determining the required fuel prior to take off? (Remember to add the answer to this question to your Physics 20 course folder.)

Module 2: Lesson 2 Assignment

Remember to submit the answer to Discuss to your teacher as part of your Lesson 2 Assignment.

Discuss

Navigational displays in a commercial jet aircraft include the following three velocities:

- air velocity, the plane’s velocity relative to the air it moves in
- wind velocity, the air’s velocity relative to the ground
- ground velocity, the plane’s velocity relative to the ground

Devices on the exterior of the plane are used to measure the air velocity. The global positioning system is used to measure the ground velocity. How is the wind’s velocity relative to the ground measured from the plane? Prepare to meet in the discussion area and work on developing a method for determining the wind velocity. You will likely use both the air velocity and ground velocity and a vector diagram in your method. Be prepared to explain what calculations must be made in order to display the wind velocity in the navigational controls.

Reflect on the Big Picture

In this lesson you learned how to add vectors using a more algebraic method. You continued working with the geometry of vectors and also used trigonometry and algebra. Did you find that drawing the addition of vectors helped you understand this algebraic approach? Did you enjoy this ”home renovation” approach of tearing things down and then rebuilding them?

To help you reflect on your learning experience in this lesson, complete at least one of the following activities:

- Create a set of study questions that would help a new learner master the concepts and math for adding two vectors numerically.

- Draw or paint a picture that contains the essence of this "home renovation" method of adding vectors. Add a short written explanation of how your picture explains this lesson's work.

Store your completed reflection in your Physics 20 course folder.

Module 2: Lesson 2 Assignment

Make sure you have completed all of the questions for the Lesson 2 Assignment. Check with your teacher about whether you should submit your assignment now or wait until all of the Module 2 assignments have been completed.

For the four sports that you researched in Lesson 1, list the ways that their equipment can be analyzed using vectors or be better understood using vector analysis. Continue to store your work in your Physics 20 course folder. You will submit this research to your teacher at the end of Lesson 4 as part of your Module 2 Project.

Lesson Summary

In this lesson you explored the following essential questions:

- How do you mathematically determine the magnitude and direction of the sum of two vectors, such as air velocity and wind velocity, for a flying plane?

- How do you calculate the components of the two vectors to be added when given the magnitudes and directions?

How do you determine the magnitude and direction of the sum of two vectors?

If two vectors $\vec{v}_1$ and $\vec{v}_2$ are given in terms of magnitude and direction, a resultant can be calculated by doing the following:

- Use a vector diagram and trigonometric functions to convert the vectors to component form.

- Add the components ($x_{\text{total}} = x_1 + x_2$) and ($y_{\text{total}} = y_1 + y_2$). Remember to include positive or negative directions.

- Draw the resultant vector using the x_{total} component and the y_{total} component. Remember to include positive or negative directions.

- Calculate the resultant magnitude using the Pythagorean theorem, $c^2 = a^2 + b^2$.

- Calculate the direction using the appropriate trigonometric function (tangent function).

Lesson Glossary

vector diagram: a diagram using vectors to show motion

Vector diagrams may instead be used to show forces, fields, or momentum.

Lesson 3—Solving Projectile Problems

Get Focused

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A trebuchet, like the one pictured here, was originally invented as a medieval siege weapon used to either batter the mason walls of a castle or to throw projectiles over the wall. Today, it is used mostly as a novelty device, as seen here in action at a pumpkin-throwing contest.

How far can such a device throw a pumpkin? Ignoring air resistance, how is the range (horizontal displacement) and the time of flight for a projectile, such as a pumpkin, determined?

As in the case of the trebuchet, you will learn that in the absence of air resistance, projectile motion of an object can be described as a combination of two motions—a horizontal motion with constant velocity and a vertical motion with uniform acceleration due to gravity.

In this lesson you will explore the following essential question:

- How do you use kinematic equations to determine the range (horizontal displacement) and the time of flight for a projectile with constant acceleration due to gravity?

Module 2: Lesson 3 Assignments

Your Lesson 3 Assignment in the Module 2 Assignment Booklet requires you to submit a response to the following:

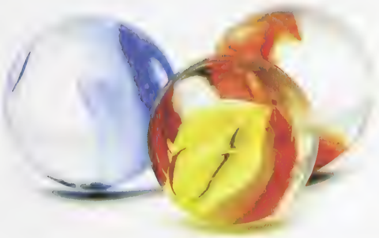
- Lab—LAB 2 and LAB 3
- Try This—TR 1, TR 2, TR 3, and TR 4

You must decide what to do with the questions that are not marked by the teacher.

Remember that these questions provide you with the practice and feedback that you need to successfully complete this course. You should respond to all the questions and place those answers in your course folder.

Explore

Imagine standing at the edge of a cliff overlooking the ocean. You have two identical marbles that you send over the edge of the cliff at exactly the same time. Marble 1 is dropped from rest at the same time that marble 2 is thrown horizontally as illustrated here.



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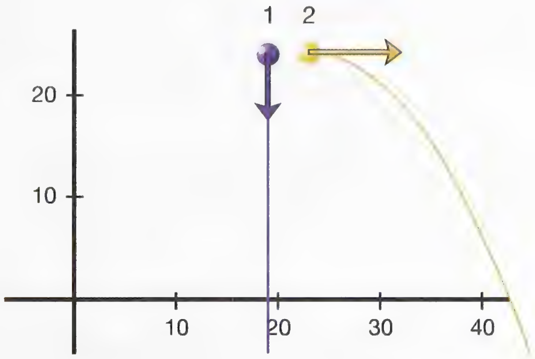


Based on the initial motion of each marble, predict which one of the following events will occur if air resistance is negligible.

- Marble 1 hits the water first.
- Marble 2 hits the water first.
- Both marbles hit the water at the same time.

It is tempting to say that marble 1 would reach the water first because it is starting to move down immediately, whereas marble 2 is starting out horizontally. The following lab will help you determine the answer by allowing you to visually observe the path of two falling objects.

Lesson 3 Lab: Projectile Motion—Two Balls

Introduction

The simulation used for this lab shows two balls performing simultaneous projectile motions. Go to **www.learnalberta.ca**. You may be required to input a username and password. Contact your teacher for this information. Enter the search terms "projectile motion" into the search bar. Choose the item called "Projectile Motion: Two Balls."

This simulation lets you see how gravity affects projectiles. You can learn more about the simulation and how to use it by reading the Show Me found at the top of the simulation screen.

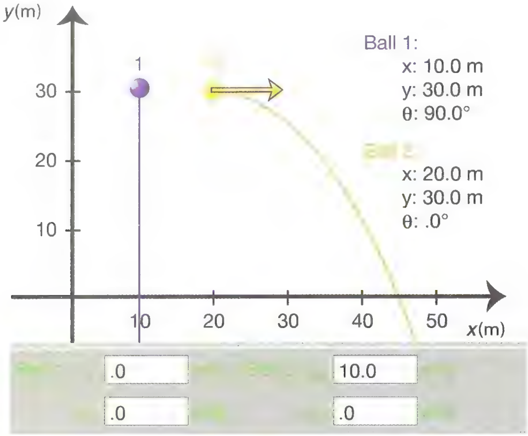
Problem

Does the horizontal velocity of a projectile affect the time that a projectile will be in the air?

Open the simulation; then continue with the procedure.

Procedure

Test your prediction using the projectile simulation.



Ball 1:
x: 10.0 m
y: 30.0 m
θ: 90.0°

x: 20.0 m
y: 30.0 m
θ: .0°

.0
.0
10.0
.0

- On the simulation, click the "Reset" button (🟩). You may have to drag the bottom border of the window down to see the labels on the buttons.
- Set the position and velocity of each ball as follows:
  - Position ball 1 at $(x, y) = (10.0, 30.0)$ m by clicking on it and dragging it to that position. Click the "Data" button (🟩) at the bottom to display the exact coordinates. You can then click and drag the left ball to the desired position.
  - Position ball 2 at $(x, y) = (20.0, 30.0)$ m.
  - Set the velocity of each ball as follows:

    ball 1 (purple): $v_x = 0.0$ m/s and $v_y = 0.0$ m/s

    ball 2 (orange): $v_x = 10.0$ m/s and $v_y = 0.0$ m/s.
  - Press "Enter."

Display the balls' paths by clicking the "Trace" button (🟩) and "Play" button (🟩). Watch the simulation to verify which ball reaches the x-axis first. Press the "Pause" button (🟩) to stop the action.

Observation and Analysis

LAB 1. Do both balls reach the x-axis at the same time? On the simulation, select "Rewind" (🟩), and set the step timer (Time step=0.2s) to 0.02 s. Select the "Play" button, and then pause it when the balls near the x-axis.

Step through the motion by repeatedly pressing the "Step" button (⏯). To display the exact coordinates, you need to have the "Data" button (▦) clicked. Record the time required for each ball to reach the *x*-axis.

a. time for ball 1 to reach the *x*-axis __________

b. time for ball 2 to reach the *x*-axis __________

Module 2: Lesson 3 Assignment

Remember to submit the answers to LAB 2 and LAB 3 to your teacher as part of your Lesson 3 Assignment in the Module 2 Assignment Booklet.

LAB 2. Do both balls have the same vertical (v_y) component of velocity when they reach the *x*-axis? On the simulation, select "Rewind." Select the "Play" button, and then pause it when the balls near the *x*-axis. Step through the motion by repeatedly pressing the "Step" button (⏯). Record the vertical velocity (v_y) of each ball when they reach the *x*-axis.

a. vertical component of velocity (v_y) when ball 1 reaches the *x*-axis ____

b. vertical component of velocity (v_y) when ball 2 reaches the *x*-axis ____

LAB 3. Will a change in the horizontal velocity affect the vertical motion of ball 2? On the simulation, select "Rewind" (⏮). Increase the horizontal velocity of ball 2 to 20.0 m/s. Step through the motion by repeatedly pressing the "Step" button (⏯). Again, record the vertical velocity (v_y) of each ball when they reach the *x*-axis.

a. vertical component of velocity (v_y) when ball 1 reaches the *x*-axis ____

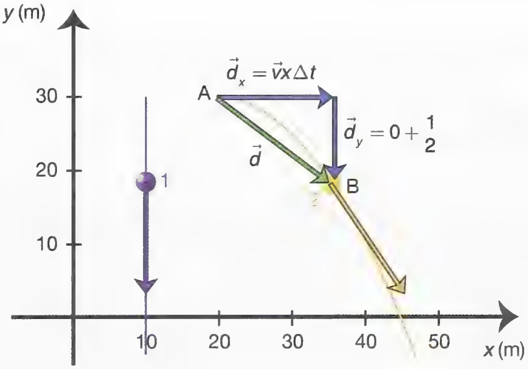
b. vertical component of velocity (v_y) when ball 2 reaches the *x*-axis ____

c. Did the vertical motion of ball 2 change? Explain why or why not.

Lab Summary

In this lab you explored whether the horizontal velocity of a projectile will affect the time that a projectile will be in the air.

The two balls reach the *x*-axis at the same time and with the same vertical component of velocity. Thus, the vertical motions are identical. You can confirm this by observing the values of the vertical position (*y*) and vertical velocity (v_y) of each ball at any time during the projectile motion. They will always be the same for the two balls. In contrast, the horizontal (*x*) components of velocity of the two balls are different. While ball 2 travels horizontally at a uniform speed, ball 1 does not move horizontally.



The motion of ball 2 is characterized by two independent motions at the same time:

- horizontal motion with constant velocity (v_x)
- vertical motion with constant acceleration (a_y)

The two motions are independent of each other. This is illustrated by the blue vectors in the image. The net displacement ($\Delta\vec{d}$) of ball 2 in going from position A to position B is the vector sum of the ball's horizontal displacement and vertical displacement. These two displacements are defined differently because one is accelerating while the other is not.

- horizontal displacement

$$\Delta d_x = v_x \Delta t$$

- vertical displacement

$$\Delta d_y = v_{iy}\Delta t + \tfrac{1}{2}a_y(\Delta t)^2$$
$$\Delta d_y = 0 + \tfrac{1}{2}a_y(\Delta t)^2$$

These two displacements are progressing simultaneously as ball 2 is moving along the orange trajectory.

As the equations for the displacements indicate, the horizontal displacement proceeds with constant velocity equal to v_x and the vertical displacement with constant acceleration equal to a_y.

Read

Read "Projectile Motion" on pages 102 to 104 of your textbook. As you read, look for the answer to the following Self-Check question.

Self-Check

SC 1. What is the main difference between the horizontal motion and the vertical motion of a projectile?

Check your work with the answer in the appendix.

Read

Read "Objects Launched Horizontally" and "Projectile Motion" on pages 104 to 108 of your textbook to see how what you have learned in this lesson is used in solving problems. As you read, look for the answer to the following Self-Check questions.

Self-Check

SC 2. For projectiles launched in the horizontal (x) direction where $a_x = 0$, what does the equation $\Delta d_x = v_{ix}\Delta t + \frac{1}{2}a_x(\Delta t)^2$ become?

SC 3. For projectiles launched in the horizontal (x) direction where the initial vertical velocity (v_{iy}) = 0, what does the equation $\Delta d_y = v_{iy}\Delta t + \frac{1}{2}a_y(\Delta t)^2$ become?

Check your work with the answer in the appendix.

Solving Projectile Problems: Calculating the Time of Fall and Horizontal Displacement (Range) of a Projectile

The following problem and solution is an example of how these projectile motion problems can be solved. Following this example, you will be asked to solve several of this type of problem for your assignment.

Question

A ball is fired horizontally at 15.0 m/s from a vertical height of 40.0 m. How long will it be in the air? How far away, horizontally, will it be when it lands (horizontal range)?

Given

vertical case
$a_y = -9.81 \text{ m/s}^2$
$v_{yi} = 0.00 \text{ m/s}$
$\Delta d_y = -40.0 \text{ m}$

horizontal case
$a_x = 0$
$v_{xi} = 15.0 \text{ m/s}$

Required

the time the ball remains in the air (Δt)
the horizontal range of the ball (Δd)

Analysis

This might seem like a difficult two-dimensional problem. However, knowing that the vertical motion of the ball is independent of the horizontal motion, the question becomes two separate one-dimensional problems:

- determining the vertical motion with constant acceleration
- determining the horizontal motion with uniform velocity

To determine the vertical motion with constant acceleration, only vertical information will be required to solve the time of fall.

To determine the horizontal motion with uniform velocity, only horizontal information will be required to solve the horizontal displacement (range).

Solution

The equation for vertical displacement can be used to calculate the time of fall.

$$\Delta d_{y} = v_{iy}\Delta t + \tfrac{1}{2}a_{y}(\Delta t)^{2}$$
$$\Delta d_{y} = 0 + \tfrac{1}{2}a_{y}(\Delta t)^{2}$$
$$\Delta d_{y} = \tfrac{1}{2}a_{y}(\Delta t)^{2}$$

$$(\Delta t)^{2} = \frac{\Delta d_{y}}{\tfrac{1}{2}a_{y}}$$
$$\Delta t = \sqrt{\frac{2\Delta d_{y}}{a_{y}}}$$
$$\Delta t = \sqrt{\frac{2(-40.0\ \text{m})}{(-9.81\ \text{m/s}^{2})}}$$
$$\Delta t = 2.856\ \text{s}$$

It takes 2.86 s for the ball to fall.

The equation for horizontal uniform motion can be used to find the range.

$$\begin{aligned}\Delta d_{x} &= v_{x}\Delta t\\ &= (15.0\ \text{m/s})(2.856\ \text{s})\\ &= 42.8\ \text{m}\end{aligned}$$

Paraphrase

The ball lands 42.8 m from the launch position.

Self-Check

SC 4. Answer practice problem 1 for "Example 2.10" on page 107 of your textbook.

Check your work with the answer in the appendix.

Module 2: Lesson 3 Assignment

Remember to submit the answers to TR 1, TR 2, TR 3, and TR 4 to your teacher as part of your Lesson 3 Assignment.

Try This

TR 1. A ball is dropped from a vertical height of 45.0 m. How long will it be in the air? Show your calculations. Verify your answer using the simulation.

TR 2. A ball is launched with a horizontal velocity of 10.0 m/s from a 20.0-m cliff. How long will it be in the air? How far will it land from the base of the cliff? Show your calculations. Verify your answer using the simulation.

TR 3. A ball is launched with a horizontal velocity of 14.0 m/s from a vertical height of 25.0 m. How long will the ball be in the air? How far, horizontally, will the ball move before hitting the ground? Show your calculations. Verify your answer using the simulation.

TR 4. An arrow is fired horizontally from the top of a 50.0-m vertical cliff and lands 120 m away. At what speed was the arrow fired? Show your calculations. Verify your answer using the simulation.

Reflect and Connect

Today, the largest working trebuchet (of medieval design) can be found at Warwick Castle in central England. It is used as a tourist attraction and can be fired by members of the public under professional supervision. It stands 19 m tall and uses a 5443-kg counterweight to fire 15-kg stone balls nearly 100 m. More modern trebuchet designs can throw pianos weighing 225 kg over 250 m. In England, a group of farmers threw a car over 100 m and a 55-gallon drum filled with gasoline over 300 m. Research the medieval and modern design of today's trebuchet.

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Reflect on the Big Picture

In this lesson you learned how to solve projectile-motion problems. You worked with the independent horizontal and vertical components of motion. You saw that Earth's constant gravity gives important information when determining the path of a projectile. Can you imagine the joy the first scientist felt when this idea formed?

To help you reflect on your learning experience in this lesson, complete at least one of the following activities:

- The ideas are fairly simple, but the calculations are not. Can you express the components of motion in a short paragraph that also shows how to solve a problem like TR 3?
- Depict projectile motion in a painting, drawing, or sculpture. Add a short written explanation of how your artwork explains this lesson's work. You can add your description to the course folder.

Store your completed reflection in your Physics 20 course folder.

Going Beyond

Principle of Projectile Motion

The projectile motion of an object subject to no air resistance is a combination of two motions—a horizontal motion with a constant velocity equal to the object's initial horizontal velocity and a vertical motion with a constant vertical acceleration (gravity).

The principle of projectile motion explains an interesting phenomenon that relates to the targeting of a falling object.

Self-Check

SC 5. Where would you aim a projectile in order to hit a falling object? Try to find the answer "experimentally" with the Projectile Motion: Two Balls simulation at **www.learnalberta.ca** by doing the following:

- Set the air resistance to 0.000 kg/m. (Adjust air resistance with “Drag Coeff. Slider.”)
- Press the “Data” button for the display of information for each balls.
- Drag ball 1 to point (x, y) = (0.0, 0.0) m.
- Drag ball 2 to any point to the right and well above ball 1.
- Aim the initial velocity of ball 1 directly at ball 2.
- Set the initial horizontal and vertical velocity of ball 2 to zero (similar to Figure 1).
- Play the motions, and observe the path of the projectiles.

SC 6. Change the magnitude (not the direction) of the initial velocity for ball 1 to any value you choose. Press “Play,” and observe for a collision. Will the two balls always collide as long as the velocity of ball 1 is directed straight towards ball 2?

Check your work with the answer in the appendix.

An explanation for the collision in the Self-Check questions is in the following equation:

$$\Delta\vec{d} = \vec{v}_i\Delta t + \tfrac{1}{2}\vec{a}(\Delta t)^2$$

This equation says that displacement is the sum of two terms, $\vec{v}_i\Delta t$ and $\frac{1}{2}\vec{a}(\Delta t)^2$. In words, this equation describes the principle of projectile motion.

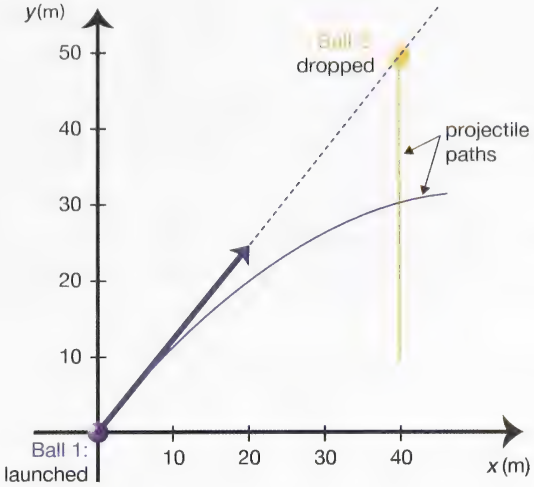

Figure 1

The projections of projectile motion onto the horizontal and vertical axes are motions with constant horizontal velocity and constant vertical acceleration, respectively. The acceleration along the vertical axis is downward and, on Earth, has a magnitude of $g = 9.81$ m/s^2.

- The horizontal displacement caused by constant horizontal velocity is defined by $\Delta\vec{d}_x = \vec{v}_x \Delta t$.
- The vertical displacement with constant vertical acceleration starting from rest is defined by

$$\Delta\vec{d}_y = \vec{v}_{iy}\Delta t + \tfrac{1}{2}\vec{a}_y(\Delta t)^2$$
$$\Delta\vec{d}_y = 0 + \tfrac{1}{2}\vec{a}_y(\Delta t)^2$$

What does the principle of projectile motion imply when ball 1 has its initial velocity aimed at ball 2? The implication is illustrated in Figure 2. The dotted line shows what the path of the projectile would be if there were no gravitational effects. The blue curved path shows the actual path of the projectile. The path curves because gravity pulls the projectile down after it is released.

First, imagine that there is no vertical acceleration acting on either ball. If this were the case, then ball 1 would travel along a straight-line path with displacement defined by $\Delta\vec{d} = \vec{v}_i\Delta t$ until it collided with ball 2, which remains at rest because there is no vertical acceleration acting on it either.

Now, imagine the same situation but with a vertical acceleration. During the time it takes ball 1 to reach the original starting position of ball 2, the vertical acceleration causes ball 1 to have a vertical displacement defined by $\Delta\vec{d} = \tfrac{1}{2}\vec{a}(\Delta t)^2$. Simultaneously, the same acceleration acts on ball 2 during the same time interval to cause exactly the same vertical displacement.

Therefore, the two balls must collide at some point. The two simultaneous displacements of ball 1 are shown in red, and the downward displacement of ball 2 is shown in yellow on Figure 2. This completes the explanation.

Notice that the explanation did not depend on the initial speed of the projectile. Only the direction of the initial velocity was critical. The initial speed of the projectile only determines how much time elapses until the two balls collide—not whether they collide.

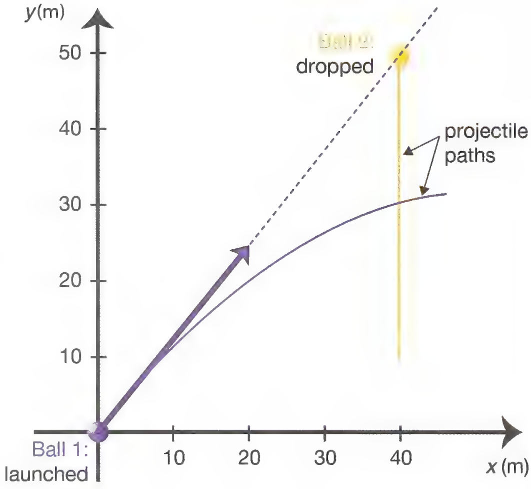

Figure 2

For example, suppose that the horizontal motion of the projectile was 20.0 m/s. Of the two motions contributing to the projectile motion of ball 1, only the horizontal motion with the constant velocity will move the ball towards the vertical fall line of ball 2. Since ball 1 has to move a horizontal distance of $\Delta d_x = 40.0$ m to reach the vertical trajectory of ball 2, the time interval Δt required is

$$\Delta t = \frac{d_x}{v_x}$$

$$\Delta t = \frac{40.0\ \text{m}}{20.0\ \text{m/s}}$$

$$\Delta t = 2.00\ \text{s, correct to 3 significant digits}$$

Module 2: Lesson 3 Assignment

Make sure you have completed all of the questions for the Lesson 3 Assignment. Check with your teacher about whether you should submit your assignment now or wait until all of the Module 2 assignments have been completed.

Select an item from your Physics 20 course folder that you haven't yet shared with your teacher, and share it.

Choose one of the four sports that you researched in Lesson 1 and Lesson 2. Do a more in-depth search for information on the requirements of the equipment used and about the standards that the equipment must meet. Continue to store your project work in your Physics 20 course folder. You will submit this research to your teacher at the end of Lesson 4 as part of your Module 2 Project.

Lesson Summary

In this lesson you explored the following essential question:

- How can the kinematic equations of motion with constant acceleration or uniform motion be used to solve projectile problems?

Projectile problems seem like difficult two-dimensional problems. However, knowing that the vertical motion of the ball is independent from the horizontal motion, the question becomes two straightforward one-dimensional problems—one for vertical motion with constant acceleration and another for horizontal motion with uniform velocity.

- Vertical information is required to solve the time of fall and vertical displacement and velocity at any time.

- Horizontal information is required to solve for the range of a projectile (horizontal displacement).

Lesson 4—Understanding and Predicting the Motion of a Projectile

Get Focused

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The game of golf involves not only the skill of hitting the ball properly, but also predicting where it will land. It is properly played using many different clubs, and each club is designed to give the ball a unique trajectory (path of flight). For example, the pitching wedge shown in this photo is designed to give the ball a trajectory that is vertically high but horizontally short. A driver used for a tee shot, on the other hand, is designed to give the ball a trajectory that is horizontally very long.

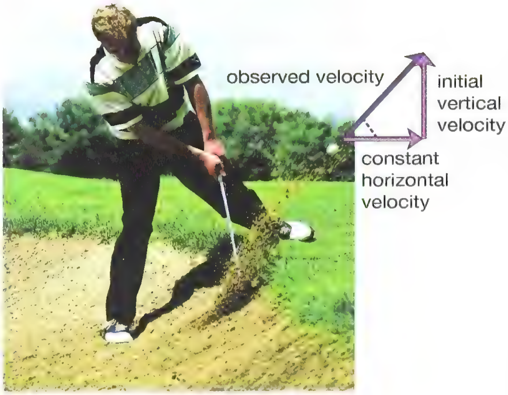
So how do you know which club to use each time you hit the ball? Well, it depends on several factors—how far you want the ball to travel horizontally (the distance to the pin in most cases); obstacles that are near or far; and environmental conditions, such as wind. Each club is designed to give the ball a unique combination of horizontal and vertical velocity.

The distance the ball travels horizontally depends on both the vertical and horizontal velocity of the ball just after it is struck by the club head.

In this lesson you will explore the following essential question:

- How can **independent** vertical and horizontal motions be analyzed with kinematic equations to predict the two-dimensional position and velocity of a projectile at various times?

independent: not affecting one another



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Module 2: Lesson 4 Assignments

Your Lesson 4 Assignment in the Module 2 Assignment Booklet requires you to submit a response to the following:

- LAB—LAB 1, LAB 2, LAB 3, and LAB 5
- Try This—TR 1

You must decide what to do with the questions that are not marked by the teacher.

Remember that these questions provide you with the practice and feedback that you need to successfully complete this course. You should respond to all the questions and place those answers in your course folder.

Explore

Projectiles Launched at an Angle

In **projectile** motion an object is thrown at an angle from the ground or launched horizontally from the edge of something, such as a table or a cliff. In earlier lessons you learned how to describe the motion of objects that were either moving horizontally or vertically. In Lesson 3, which you just completed, you considered motion where the object was launched horizontally. In this lesson you will use kinematic equations to predict the motion of objects launched at an angle to the horizontal. For any projectile, the vertical motion is independent of the horizontal motion. In this case, the vertical component of motion is being affected by the acceleration due to gravity while the horizontal component is not. The result is that the object follows a **parabolic** path or trajectory. A parabolic path results because the projectile is falling due to the attraction of gravity and, at the same time, it is moving horizontally at a constant rate.

projectile: a moving object acted on by no forces other than gravity

parabolic an arched shape having the form of a parabola

Watch and Listen—Analyzing the Motion of a Projectile

Go to your Physics 20 Multimedia DVD, and work through the "Projectile Motion" animation to improve your understanding of projectile motion.

Follow the directions as they appear. Use the forward and back arrows, as needed, to proceed through the animation as many times as you like.

Lesson 4 Lab: Projectile Motion—One Ball Simulation

Introduction

The simulation used in this lab shows the path of a ball being thrown on Earth. This is a simulation of the projectile motion of a ball without air resistance, where the acceleration is only due to gravity. You can learn more about the simulation and how to use it by reading the Show Me found at the top of the simulation screen.

Problem

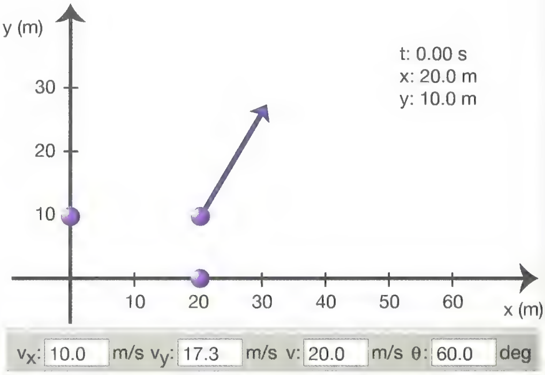
Are the vertical and horizontal motions of a projectile independent from one another?

Go to **www.learnalberta.ca**. You may be required to input a username and password. Contact your teacher for this information. Enter the search terms "projectile motion" in the search bar. Choose the item called "Projectile Motion: One Ball."Open the simulation; then continue with the procedure.

Procedure

The simulation will be used to show the motion of a simple projectile with no air resistance.

- Click the "Reset" button (⟳).
- Position the ball at $(x, y) = (20.0 \text{ m}, 10.0 \text{ m})$ by clicking on it and dragging it to that position. Click the "Data" button to see your coordinates displayed.



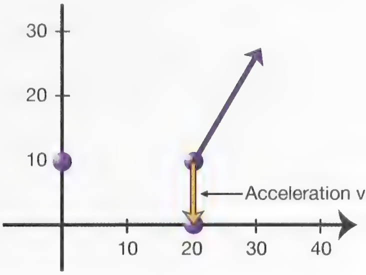
t: 0.00 s
x: 20.0 m
y: 10.0 m

v_x: 10.0 m/s v_y: 17.3 m/s v: 20.0 m/s θ: 60.0 deg

Figure 1

- Set the initial velocity to a magnitude of 20.0 m/s (in the third box at the bottom of the screen). Direct the velocity at an angle (θ) of 60.0° relative to the positive x-axis (in the fourth box at the bottom of the screen). Press "Enter" on your keyboard. The simulation should be configured as shown in Figure 1.
- Click "Play" (▶). The ball will be moving without air resistance. View the motion several times by clicking the "Rewind" button (⟲).
- Click the "Vectors" button (vector), and observe the acceleration vector by selecting the acceleration square. Sketch and label the acceleration vector on Figure 1.
- Click "Play" (▶). View the motion several times by clicking the "Rewind" button (⟲).

Acceleration Vector Sketch from Procedure



Observation and Analysis

Module 2: Lesson 4 Assignment

Remember to submit the answers to LAB 1, LAB 2, and LAB 3 to your teacher as part of your Lesson 4 Assignment.

LAB 1. What is the ball's acceleration on the way up, at the top, and on the way down?

____________ m/s2

You should be able to observe that the acceleration vector remains constant throughout the motion, both in magnitude and direction. In particular, it is not zero at the peak of the motion.

LAB 2. Click "Rewind" (), and change the ball's initial velocity to a magnitude of 40.0 m/s and to a direction that forms an angle (θ) with the positive x-axis equal to 50.0°. Press "Play," and observe the acceleration vector. Did the acceleration vector change when you entered the new initial velocity, and does it vary during the new motion? Explain why the answers to both questions are the same.

LAB 3. Change the ball's initial velocity to a magnitude of 20.0 m/s and to a direction that forms an angle (θ) with the positive x-axis equal to 90.0°. Press "Play," and observe the straight up-and-down motion.

a. Does the acceleration ever become zero?

b. Does the velocity ever become zero? If so, where in the projectile path does this occur?

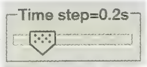
c. Does a zero velocity imply that no acceleration is acting? Explain.

LAB 4. Position the ball at (x, y) = (20.0 m, 10.0 m) and set the initial velocity to a magnitude of 20.0 m/s. Direct the velocity at an angle (θ) of 60.0° relative to the positive x-axis, and press "Play". Observe the horizontal velocity component (v_x). Does it change throughout the motion? Based on this observation, is there a horizontal acceleration? Remember to add this answer to your course folder.

Hint: The shadow ball on the horizontal axis moves with constant velocity. Thus, there is no horizontal acceleration. However, there is vertical acceleration. The shadow ball on the vertical axis moves up and then down and does not move with constant velocity.

Module 2: Lesson 4 Assignment

Remember to submit the answer to LAB 5 to your teacher as part of your Lesson 4 Assignment.

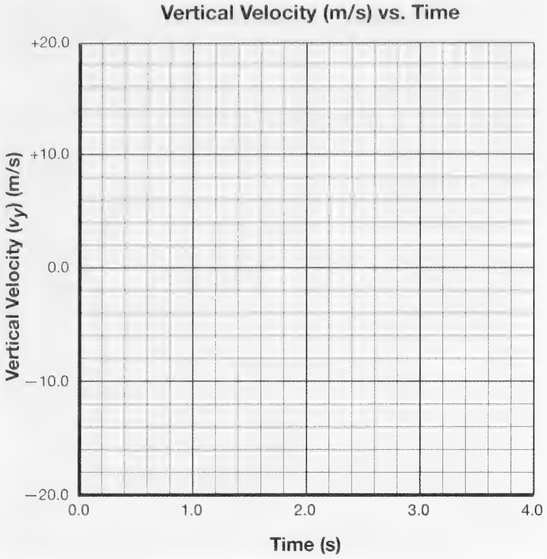
LAB 5. Determine the vertical acceleration by plotting and calculating the slope of the vertical velocity-time graph. Set the step slider (

) to a time step of size 0.50 s. If you can't get the slider to drag to the exact value, click the button (#) next to the step slider and type 0.50 s. Then press "Rewind" (↺).

a. Step through the motion by repeatedly pressing the "Step" button (⏭), and record the v_y component of the velocity in Table 1.

t (s)	v_y (m/s)
0.0	______
0.5	______
1.0	______
1.5	______
2.0	______
2.5	______
3.0	______
3.5	______

b. Plot vertical velocity (v_y) versus time on the following graph.



c. Calculate the slope of the line for the vertical velocity-time graph. Include the units for velocity and time, and, using the units, prove that the slope must be equal to the vertical acceleration.

Lab Summary

The ball's acceleration in the x direction is 0 m/s^2, and in the y direction it is –9.81 m/s^2. This is written using component notation as follows:

$a_x = 0\text{ m/s}^2$, $a_y = -9.81\text{ m/s}^2$

The vertical acceleration is called the acceleration due to gravity because it is caused by gravity alone. This result allows the following description of projectile motion in terms of the motions of two shadow balls (one moving on the horizontal axes and another moving on the vertical axes).

The motion of the horizontal shadow ball is motion with constant velocity.

The motion of the vertical shadow ball is motion with constant acceleration. The acceleration of the shadow ball moving on the vertical axis is downward and, on earth, has magnitude $g = -9.81\text{ m/s}^2$.

Predicting a Projectile's Motion

The displacement and velocity of a projectile at any time during its flight may be determined in advance. All you would need to know are the **initial velocity** and initial position of the projectile.

initial velocity: the velocity of an object when an experiment begins

The next example shows how you can use just the initial velocity and initial position to calculate the displacement and velocity a projectile will have at some later time.

Suppose a ball positioned at $(x, y) = (20.0\text{ m}, 10.0\text{ m})$ has an initial velocity of 20.0 m/s at an angle (θ) of 60.0° relative to the positive x-axis. That is, $(v_i, \theta) = (20.0\text{ m/s}, 60.0°)$.

What is the ball's velocity when $t = 2.50$ s, and what will the ball's displacement be during this time interval?

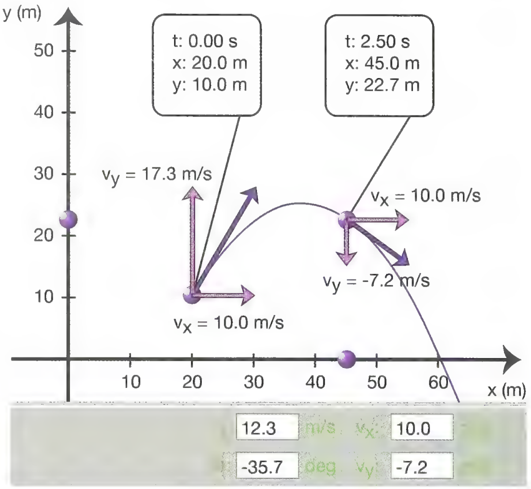


Figure 1

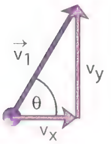
Remember that the horizontal and vertical components of projectile motion are independent of each other. The equations for the x components do not contain any y components, and vice versa. This simplifies the mathematical aspects of projectile motion a great deal. Instead of a complicated two-dimensional problem, you are dealing with two separate one-dimensional problems.

The kinematic equations for motion with constant acceleration that you studied in Module 1 will be required to calculate both the velocity and displacement.

Those equations are given in terms of x and y components (one-dimensional). The initial position is already in this form, but the initial velocity is in polar form and needs to be converted to component form, effectively separating the x and y components of the given velocity.

These are the calculations:

- horizontal component of the initial velocity



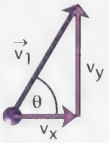
$$\cos\theta = \frac{v_x}{v_i}$$

$$v_x = \cos(\theta)(v_i)$$

$$v_x = \cos(60°)(20.0\text{ m/s})$$

$$v_x = 10.0\text{ m/s}$$

- vertical component of the initial velocity



$$\sin\theta = \frac{v_y}{v_i}$$
$$v_y = \sin(\theta)(v_i)$$
$$v_y = \sin(60°)(20.0\ \text{m/s})$$
$$v_y \doteq 17.320\,508\,08\ \text{m/s}$$
$$v_y = 17.3\ \text{m/s, correct to 3 significant digits}$$

Note that these values are shown in the data panel before the simulation is played.

Calculating Velocity

Substitution of the initial velocity components and the time ($t = 2.50$ s) into the acceleration equation gives the final velocity of the x and y components at the time specified in the question.

- horizontal component of velocity at $t = 2.50$ s

$$a_x = \frac{v_{fx} - v_{ix}}{\Delta t}$$
$$v_{fx} = v_{ix} + a_x \Delta t$$
$$v_{fx} = (+10.0\ \text{m/s}) + (0\ \text{m/s}^2)(2.50\ \text{s})$$
$$v_x = +10.0\ \text{m/s, correct to 3 significant digits}$$

- vertical component of velocity at $t = 2.50$ s

$$a_y = \frac{v_{fy} - v_{iy}}{\Delta t}$$
$$v_{fy} = v_{iy} + a_y \Delta t$$
$$v_{fy} = (+17.3\ \text{m/s}) + (-9.81\ \text{m/s}^2)(2.50\ \text{s})$$
$$v_y \doteq -7.225\ \text{m/s}$$
$$v_y = -7.23\ \text{m/s, correct to 3 significant digits}$$

Calculating Displacement

Substitution of the initial velocity components and the acceleration components into the equation for displacement when $t = 2.50$ s gives

- horizontal component of displacement when $t = 2.50$ s

$$d_x = v_{ix}\Delta t + \tfrac{1}{2}a_x(\Delta t)^2$$
$$d_x = (+10.0\ \text{m/s})(2.50\ \text{s}) + \tfrac{1}{2}(0\ \text{m/s}^2)(2.50\ \text{s})^2$$
$$d_x = 25.0\ \text{m, correct to 3 significant digits}$$

The horizontal displacement is 25.0 m. Given the initial horizontal position of 20.0 m, the horizontal position after 2.50 s will be 20.0 m + 25.0 m = 45.0 m

This agrees with the position displayed in Figure 1.

- vertical component of displacement when $t = 2.50$ s

$$d_y = v_{iy}\Delta t + \tfrac{1}{2}a_y(\Delta t)^2$$
$$d_y = (+17.3\ \text{m/s})(2.50\ \text{s}) + \tfrac{1}{2}(-9.81\ \text{m/s}^2)(2.50\ \text{s})^2$$
$$d_y = +12.593\,75\ \text{m}$$
$$d_y = +12.6\ \text{m, correct to 3 significant digits}$$

The vertical displacement is +12.6 m. Given the initial vertical position of 10 m, the vertical position after 2.5 s will be 10 m + 12.6 m = 22.6 m.

This is very close to the position displayed in Figure 1. You could expect some difference because you used 9.81 m/s^2 for the magnitude of the acceleration while the simulation uses 9.8 m/s^2.

Read

Read "Objects Launched at an Angle" on pages 108 to 110 of your textbook. As you read, look for the answers to the next Self-Check question.

Self-Check

SC 1.

a. If a projectile returns to the same height from which it was launched, what is the Δd_y?

b. Knowing the answer to SC 1.a., what formula can you use to find Δt?

SC 2. Re-open the "Projectile Motion: One Ball" simulation at **www.learalberta.ca**. Position the ball at $(x, y) = (10.0 \text{ m}, 0.0 \text{ m})$, and set the initial velocity as 25.0 m/s at an angle (θ) of 50.0° relative to the positive x-axis.

$(x, y) = (10.0 \text{ m}, 0.0 \text{ m})$ $(v_i, \theta) = (25.0 \text{ m/s}, 50.0°)$

Using the appropriate kinematics equations and initial values, calculate the position and velocity at $t = 3.00$ s. After you have completed your calculations, verify your answers by entering the launch speed and direction in the simulation and stepping through the motion to 3.00 s.

a. horizontal component of the initial velocity
b. vertical component of the initial velocity
c. horizontal velocity at 3.00 s
d. vertical velocity at 3.00 s
e. horizontal position at 3.00 s
f. vertical position at 3.00 s

Check your work with the answer in the appendix.

Module 2: Lesson 4 Assignment

Remember to submit the answer to TR 1 to your teacher as part of your Lesson 4 Assignment.

Try This

TR 1. Using the "Projectile Motion: One Ball" simulation, position the ball at $(x, y) = (0.0 \text{ m}, 0.0 \text{ m})$ and set the initial velocity as 25.0 m/s at an angle (θ) of 65.0° relative to the positive x-axis.

$(x, y) = (0.0 \text{ m}, 0.0 \text{ m})$ $(vi, \theta) = (25.0 \text{ m/s}, 65.0°)$

Using the appropriate kinematics equations and initial values, determine the position and velocity at $t = 2.40$ s. Enter the launch speed and direction on the simulation, and step through the motion to 2.40 s to verify your answers.

a. horizontal component of the initial velocity
b. vertical component of the initial velocity
c. horizontal velocity at 2.40 s
d. vertical velocity at 2.40 s
e. horizontal position at 2.40 s
f. vertical position at 2.40 s

Reflect and Connect

The physics of golf is complex. You have seen that the initial vertical and horizontal velocities of the golf ball determine its position, given a specific time interval when air resistance and environmental conditions are ignored. You also know that the horizontal and vertical motions are independent in nature, which allows you to solve what would be a complex two-dimensional problem using simpler one-dimensional solutions. In a real game of golf, there is more to consider. The ball, the club, the angle of the club face, the swing, the ball-club interaction, and the ball-air interactions all introduce new variables, such as lift. Research the physics of golf to identify and describe these other variables and how they play out in the game of golf. Given this new information, what combination of variables will maximize the range of a golf ball?

To explore this question further, use a search engine with the search words *physics of golf*.

Store your reflection in the course folder.

Reflect on the Big Picture

In this lesson you learned that using the components of a vector can give you a simpler way to analyze motion. You again worked with the geometry of vectors, but this time you broke one vector into horizontal and vertical components. You saw that you could treat each component separately and apply the kinematics equations to each component separately.

To help you reflect on your learning experience in this lesson, complete at least one of the following three activities:

- The material of this lesson is in part simple (the concept) and in part complex (the calculations). Poets often work very hard to convey complex ideas in simple language. Can you create a poem that describes the complex nature of projectile motion?

- Draw or paint a picture that shows working with the components of vectors like you did in this lesson. Add a short explanation of how your picture explains the independent nature of the horizontal and vertical component of vectors.

- What do you think you learned in this lesson? Look over the lesson, and fill in a learning log like the one that follows.

Learning Log
Describe things that you know now that you didn't know before.
Describe things that you still need to work on.
Describe ways to increase your skills and knowledge.

Going Beyond

The term *projectile motion* applies to the motion of an object in a uniform gravitational field. Such a field exists to a very good approximation within a volume extending a few kilometres horizontally and vertically from a given point on Earth's surface. Motions extending over a larger range, in which Earth's curvature and the decrease of the value of (g) with elevation become significant, are called *ballistic motions*. Compare and contrast the trajectory of these two types of motion. The projectile simulation lets you vary the magnitude of the acceleration due to gravity by means of a slider. Use the simulation to predict, observe, and explain why the maximum height changes when the value of (g) changes.

Module 2: Lesson 4 Assignment

Make sure you have completed all of the questions for the Lesson 4 Assignment. Submit your completed Module 2 Assignment Booklet to your teacher.

Your Module Project includes sending four parts to your teacher. The first three parts are the work that you completed on the project in Lessons 1, 2, and 3. The fourth part is a presentation using the information you have found.

Complete instructions follow.

Module 2 Project

You can choose to make your presentation in any form you wish. However, if you wish to use a form other than a speech (about 2 minutes long), a letter or report (about 500 words), or multimedia presentation (about 2 minutes long), you should discuss it with your teacher before beginning.

In the first three lessons of Module 2 you researched and thought about equipment used in different sports. Your final presentation should organize this information and present it in a coherent fashion. You should explain concisely how dynamics has contributed to changes in sports. Look at the scoring rubric to see how you will be graded.

Scoring Rubric for the Presentation

1. **Poor:** The presentation is irrelevant or so scant, overgeneralized, or inaccurate that it indicates a poor or almost complete lack of understanding. It contains major errors or is largely inappropriate.

2. **Limited:** The presentation is unfocused or inappropriate, potentially relevant but contains inaccuracies or extraneous detail. It shows an understanding that is confused or oversimplified.

3. **Satisfactory:** The presentation is largely relevant but may contain some minor factual errors, or there may be a mixture of relevant and extraneous information. It shows a general understanding of dynamics as it relates to sports.

4. **Proficient:** The presentation is purposeful, usually specific, relevant, and accurate, indicating a solid understanding of how dynamics affects sports.

5. **Excellent:** The presentation is comprehensive, specific, relevant, and accurate, and it demonstrates a superior understanding of how dynamics affects sports.

Each of the preliminary parts of the project will be graded on a 2-point scale: 0 points if the part shows little or no effort expended, 1 point for some effort, and 2 points if considerable effort is shown.

Lesson Summary

In this lesson you explored the following essential question:

- How can independent vertical and horizontal motions be analyzed with kinematic equations to predict the two-dimensional position and velocity of a projectile at various times?

Principle of Projectile Motion

The projectile motion of an object subject to no air resistance is a combination of two motions: a horizontal motion with a constant velocity equal to the object's initial horizontal velocity and a vertical motion with a constant vertical acceleration (gravity).

For projectile motion, when an object is thrown at an angle from the ground (Figure A) or projected horizontally off of a table or a cliff (Figure B), the principle of combining independent motions applies. The vertical component of motion is being affected by the acceleration of gravity, while the horizontal component is not affected by gravity. The result is that the object follows a parabolic path or trajectory. A parabolic path results because the projectile is falling due to the attraction of gravity and, at the same time, it is moving horizontally with constant velocity.

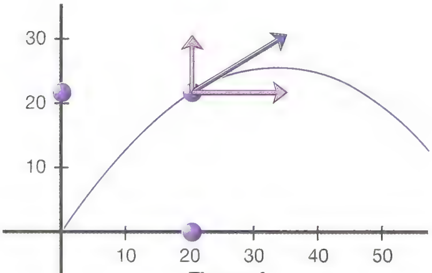


Figure A

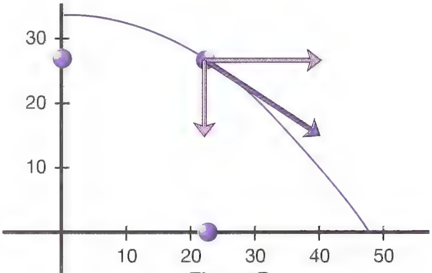


Figure B

Lesson Glossary

independent: not affecting one another

initial velocity: the velocity of an object when an experiment begins

parabolic: an arched shape having the form of a parabola

projectile: a moving object acted on by no forces other than gravity

Module 2—Motion in Two Dimensions

Module Summary

When you began this module, you were presented with the following questions:
- How do vectors help explain two-dimensional motion?
- In what ways can data be used to distinguish differing types of motion?
- In what ways can mathematics be used to help understand motion?
- How can observations and questions help explain what is seen in everyday life?

Each question is related to the description of motion or the analysis of motion. Do you see how description and analysis are just two sides of one coin? Reflect on the readings, labs, and questions you've been asked to complete in this module. You described and analyzed the motion of boats, jetliners, and other objects. You also examined how marbles and golf balls move when they travel as projectiles. This gave you greater understanding of acceleration, velocity, and displacement for objects moving in two dimensions.

Module Assessment

Module 2 Project

You can choose to make your presentation in any form you wish. However, if you wish to use a form other than a speech (about 2 minutes), a letter or report (about 500 words), or multimedia presentation (about 2 minutes), you should discuss it with your teacher before beginning.

In the first three lessons of Module 2 you researched and thought about equipment used in different sports. Your final presentation should organize this information and present it in a coherent fashion. You should explain concisely how dynamics has contributed to changes in sports. Look at the scoring rubric to see how you will be graded.

Scoring Rubric for the Presentation

Score	Criteria
5 **Excellent**	The presentation is comprehensive, specific, relevant, and accurate, and it demonstrates a superior understanding of how dynamics affects sports.
4 **Good**	The presentation is purposeful, usually specific, relevant, and accurate, indicating a solid understanding of how dynamics affects sports.
3 **Satisfactory**	The presentation is largely relevant but may contain some minor factual errors, or there may be a mixture of relevant and extraneous information. It shows a general understanding of dynamics as it relates to sports.
2 **Limited**	The presentation is unfocused or inappropriate, potentially relevant but contains inaccuracies or extraneous detail. It shows an understanding that is confused or oversimplified.
1 **Poor**	The presentation is irrelevant or so scant, overgeneralized, or inaccurate that it indicates a poor or almost complete lack of understanding. It contains major errors or is largely inappropriate.
0 **Insufficient**	The response is incomplete and/or totally off topic.

Each of the preliminary parts of the project will be graded on a 2-point scale: 0 points if the part shows little or no effort expended, 1 point for some effort, and 2 points if considerable effort is shown.

Module 2 Glossary

independent: not affecting one another

initial velocity: the velocity of an object when an experiment begins

parabolic: an arched shape having the form of a parabola

parallelogram method: a method of adding vectors, where the two vectors' tails start at the same point and define adjacent sides of a parallelogram

The diagonal of the parallelogram that starts where the tails meet is the resultant.

projectile: a moving object acted on by no forces other than gravity

resultant: a vector sum of two or more other vectors

tip-to-tail method: a method of adding vectors, where the second vector begins at the end (tip) of the first vector

The resultant is the vector from the tail of the first vector to the tip of the second.

vector diagram: a diagram using vectors to show motion

Unit A Summary

In this unit you gained an understanding of displacement, velocity, and acceleration. You have compared and contrasted scalar and vector quantities and explained uniform and uniformly accelerated motion. You are now able to interpret the motion of one object relative to another using displacement and velocity vectors and can explain two-dimensional motion in a horizontal or vertical plane using vector components.

From this unit, you have identified common applications of kinematics, such as the average speed of a car trip. You used the new language you acquired as you investigated the application of kinematic principles, such as determining the appropriate length of airport runways or the timing of traffic lights.

The skills that you used in Unit A will continue to be developed in Unit B—Dynamics.

Unit A Assessment

A student standing on the top of a cliff shoots an arrow from a height of 30.0 m at 25.0 m/s and an initial angle of 32.0° above the horizontal. Show all your work in calculating the answers to the following questions. You may check your work using any of the simulations you have previously used in Unit A.

a. What will be the horizontal and vertical components of the arrow's initial speed?

b. How high above the landscape under the cliff will the arrow rise? Assume a level landscape.

c. How far off course will the arrow be after 3.35 s if there is a 12.0 m/s crosswind?

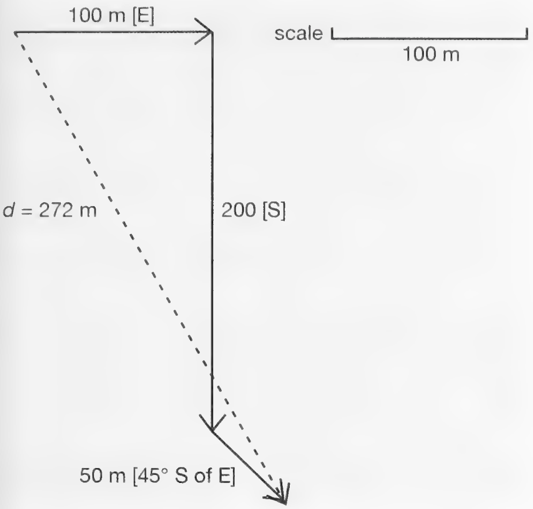
d. What will be the speed and angle of flight of the arrow 3.35 s after release if there is no crosswind?

Submit your answers to the Unit A Assignment to your teacher for marks.

Self-Check Answer

Lesson 1

SC 1.



The student's final position is 272 m [60° S of E].

SC 2.

a. The magnitude and direction of each vector $\vec{a}$ in the bottom row is the same as the vector $\vec{a}$ in the table just above it.

b. The magnitude and direction of each vector $\vec{b}$ in the bottom row is the same as the vector $\vec{b}$ in the table just above it.

c. The magnitude and direction of the resultant vector in the bottom row is the same as the magnitude and direction of the resultant vector in the table just above it.

d. The magnitude and direction of the resultant vector is the same with either method.

e. Choices and reasons will vary. The text only uses the tip-to-tail method, which works for both collinear and non-collinear vectors.

Lesson 2

SC 1.

a. blue $\vec{v}_2$

b. red $\vec{v}_1$

c. 128°

d. 52° N of W

SC 2.

Given

$\vec{v}_1 = 10$ m/s [245°]

Required

the x component (v_x)
the y component (v_y)

Analysis and Solution

The angle (θ) between the x-axis and the velocity vector is (245° – 180°) = 65°.

The x component and the y component will both be negative.

$$\cos\theta = \frac{R_x}{R}$$
$$R_x = R\cos\theta$$

$$\begin{aligned} v_x &= -(10\text{ m/s})(\cos 65°) \\ &= 4.2\text{ m/s} \end{aligned}$$

$$\sin\theta = \frac{R_y}{R}$$
$$R_y = R\sin\theta$$

$$v_x = -(10\text{ m/s})(\sin 65°)$$

$$= -9.1\text{ m/s}$$

Paraphrase

The x component of the cyclist's velocity is –4.2 m/s, and the y component of the cyclist's velocity is –9.1 m/s.

SC 3.

Given

$\vec{d}_1 = 15\text{ m }[15° \text{ N of E}]$

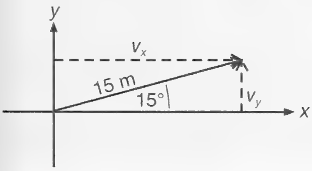
$\vec{d}_2 = 13\text{ m }[5° \text{ W of N}]$

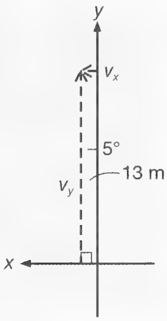
Required

the displacement of the soccer player ($\Delta\vec{d}$)

Analysis and Solution

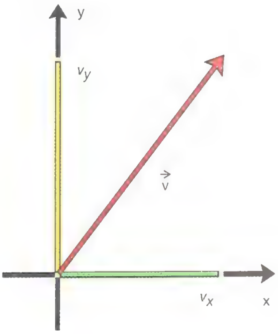
Let the x-axis represent the east direction and the y direction be the north direction. Determine the x and y components of each vector, and add the x and y components separately. Then find the magnitude of the resultant using the theorem of Pythagoras. Finally, find the angle of the resultant using the tangent function.





In the second diagram, the angle between the x-axis and the vector is (90° – 5°) = 85°.

	x direction	*y* direction
d_1	$d_{1_x} = (15\text{ m})(\cos 15°)$ $= 14.49\text{ m}$	$d_{1_y} = (15\text{ m})(\sin 15°)$ $= 3.882\text{ m}$
d_2	$d_{2_x} = -(13\text{ m})(\cos 85°)$ $= 1.133\text{ m}$	$d_{2_y} = (13\text{ m})(\sin 85°)$ $= 12.95\text{ m}$
Sum	$d_x = 13.36\text{ m}$	$d_y = 16.83\text{ m}$



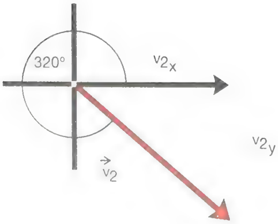
$$d = \sqrt{(13.36\text{ m})^2 + (16.83\text{ m})^2}$$
$$d = 21.49\text{ m}$$
$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$
$$\tan\theta = \frac{16.83\text{ m}}{13.36\text{ m}}$$
$$\tan\theta = 1.260$$
$$\theta = \tan^{-1}(1.260)$$
$$= 52°$$

Paraphrase

The displacement of the soccer player is 21 m [52° N of E].

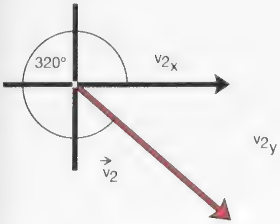
SC 4.

a.



$$\cos\theta = \frac{v_{1x}}{\vec{v}_1} \qquad \sin\theta = \frac{v_{1y}}{\vec{v}_1}$$
$$v_{1x} = \cos\theta(\vec{v}_1) \qquad v_{1y} = \sin\theta(\vec{v}_1)$$
$$v_{1x} = \cos 45°(185) \qquad v_{1y} = \sin 45°(185)$$
$$v_{1x} = +130.81 \qquad v_{1y} = +130.81$$

b.

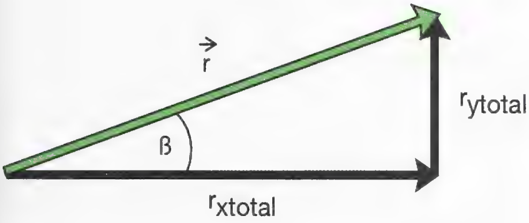


$$\cos\theta = \frac{v_{2x}}{\vec{v}_2} \qquad \sin\theta = \frac{v_{2y}}{\vec{v}_2}$$

$$v_{2x} = \cos\theta(\vec{v}_2) \qquad v_{2y} = \sin\theta(\vec{v}_2)$$

$$v_{2x} = \cos 320°(95) \qquad v_{2y} = \sin 320°(95)$$

$$v_{2x} = +72.77 \qquad v_{2y} = -61.06$$



c. $r_{xtotal} = v_{1x} + v_{2x} = (+130.81) + (+72.77) = +203.5$

d. $r_{ytotal} = v_{1y} + v_{2y} = (+130.81) + (-61.06) = +69.75$

e. $r^2 = r_x^{\ 2} + r_y^{\ 2}$

$$r = \sqrt{r_x^{\ 2} + r_y^{\ 2}}$$

$$r = \sqrt{203.58^2 + 69.75^2}$$

$$r = 215.20$$

The magnitude of the resultant vector is 215 m/s.

$$\tan\theta = \frac{|r_{ytotal}|}{|r_{xtotal}|}$$

$$\theta = \tan^{-1}\left(\frac{69.75}{203.58}\right)$$

$$\theta = 18.91°$$

f. The direction of the resultant is 18.9°.

SC 5. The velocity of the jet will be (725 km/h – 70 km/h) [W] = 655 km/h [W].

SC. 6

Given

$$\vec{v}_{\text{air}} = 400 \text{ km/h } [\text{s}]$$
$$\vec{v}_{\text{wind}} = 86.0 \text{ km/h } [\text{w}]$$

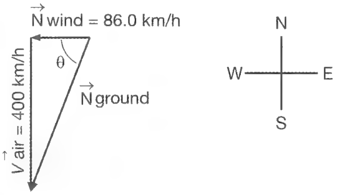
Required

a. actual velocity of plane ($\vec{v}_{\text{ground}}$)
b. heading to fly directly south (θ)

Analysis and Solution

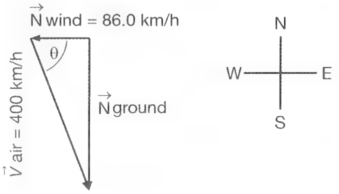
Designate south and west as positive directions.

$$\text{a. } v_{ground} = \sqrt{(v_{air})^2 + (v_{wind})^2}$$
$$= \sqrt{(400 \text{ km/h})^2 + (86.0 \text{ km/h})^2}$$
$$= 409 \text{ km/h}$$



$$\tan = \frac{\text{opposite}}{\text{adjacent}}$$
$$\tan\theta = \frac{400 \text{ km/h}}{860 \text{ km/h}}$$
$$= 4.651$$
$$\theta = \tan^{-1}(4.651)$$
$$= 77.9° [\text{S of W}]$$

b. The resultant is the ground velocity of the airplane.



$$\cos = \frac{\text{adjacent}}{\text{hypotenuse}}$$
$$\cos\theta = \frac{86.0 \text{ km/h}}{400 \text{ km/h}}$$
$$= 0.215$$
$$\theta = \cos^{-1}(0.215)$$
$$= 77.6° \ [\text{S of E}]$$

Paraphrase

a. The actual velocity of the plane is 409 km/h [77.9° S of W].
b. The heading of the plane needed to fly directly south is 77.6° S of E.

Lesson 3

SC 1. The horizontal motion of a projectile is uniform motion, where the horizontal velocity component remains constant. On the other hand, the vertical motion is accelerated motion, with the acceleration due to gravity remaining constant.

SC 2. The equation simplifies to $\Delta d_x = v_{ix}\,\Delta t$.

SC 3. The equation simplifies to $\Delta d_y = \frac{1}{2}a_y(\Delta t)^2$.

SC 4.

Given

vertical case
$a_y = -9.81\ \text{m/s}^2$
$v_{yi} = 0.00\ \text{m/s}$
$\Delta d_y = -1.25\ \text{m}$

horizontal case
$a_x = 0$
$v_{xi} = 30\ \text{cm/s}$

Required

the horizontal range of the coin (Δd)

Analysis and Solution

Even though the time that the coin remains in the air (Δt) is not asked for, solve for it first so that the range can be determined.

$$\Delta d_y = v_{iy}\Delta t + \tfrac{1}{2}a_y(\Delta t)^2$$
$$\Delta d_y = 0 + \tfrac{1}{2}a_y(\Delta t)^2$$

$$(\Delta t)^2 = \frac{\Delta d_y}{\frac{1}{2}a_y}$$
$$\Delta t = \sqrt{\frac{2\Delta d_y}{a_y}}$$
$$\Delta t = \sqrt{\frac{2(-1.25\ \text{m})}{(-9.81\ \text{m/s}^2)}}$$
$$\Delta t = 0.505\ \text{s}$$

It takes 0.505 s for the coin to fall.

The equation for horizontal uniform motion can be used to find the range.

$$\begin{aligned}\Delta d_x &= v_x \Delta t \\ &= (30\text{ cm/s})(0.505\text{ s}) \\ &= 15\text{ cm}\end{aligned}$$

Paraphrase

The coin lands 15 cm from the base of the table.

SC 5. It is tempting to think that, with (g) at 9.81 m/s^2, the velocity direction will not lead to a collision between the balls, because ball 2 will move away from the spot at which the velocity is aimed. However, you will find that with this initial velocity the balls will in fact collide. They will collide at the point in Figure 1 where the curved maroon path intersects the straight orange path.

SC 6. The initial velocity doesn't matter. The balls always collide.

Lesson 4

SC 1.

a. The Δd_y is 0.

b. The formula to use is $\Delta d_y = v_{i_y} \Delta t + \frac{1}{2} a_y (\Delta t)^2$, which reduces down to $\Delta t = \frac{-2v_i}{a}$.

SC 2.

a.

$$\cos\theta = \frac{v_x}{v_i}$$

$$\begin{aligned}v_x &= \cos\theta\left(v_i\right) \\ v_x &= \cos 50°\left(25.0\text{ m/s}\right) \\ v_x &= +16.07\text{ m/s} \\ v_x &= +16.1\text{ m/s}\end{aligned}$$

b.

$$\sin\theta = \frac{v_y}{v_i}$$

$$\begin{aligned}v_y &= \sin\theta\left(v_i\right) \\ v_y &= \sin 50°\left(25\text{ m/s}\right) \\ v_y &= +19.15\text{ m/s} \\ v_y &= +19.2\text{ m/s}\end{aligned}$$

c.

$$
\begin{aligned}
a_x &= \frac{v_{fx} - v_{ix}}{\Delta t} \\
v_{fx} &= v_{ix} + a_x \Delta t \\
v_{fx} &= (+16.07\ \text{m/s}) + \left(0\ \text{m/s}^2\right)(3.00\ \text{s}) \\
v_{fx} &= +16.07\ \text{m/s} \\
v_{fx} &= +16.1\ \text{m/s}
\end{aligned}
$$

d.

$$
\begin{aligned}
a_y &= \frac{v_{fy} - v_{iy}}{\Delta t} \\
v_{fy} &= v_{iy} + a_y \Delta t \\
v_{fy} &= (+19.5\ \text{m/s}) + \left(-9.81\ \text{m/s}^2\right)(3.00\ \text{s}) \\
v_{fy} &\doteq -10.28\ \text{m/s} \\
v_{fy} &= -10.3\ \text{m/s}
\end{aligned}
$$

e.

$$
\begin{aligned}
d_x &= v_{ix} t + \frac{1}{2} a_x t^2 \\
d_x &= (+16.07\ \text{m/s})(3.00\ \text{s}) + \frac{1}{2}\left(0\ \text{m/s}^2\right)(3.00\ \text{s})^2 \\
d_x &= +48.21\ \text{m} \\
\text{position} &= 10.0 + 48.21\ \text{m} = 58.21\ \text{m} \\
&= 58.2\ \text{m}
\end{aligned}
$$

f.

$$
\begin{aligned}
d_y &= v_{iy} t + \frac{1}{2} a_y t^2 \\
d_y &= (+19.15\ \text{m/s})(3.00\ \text{s}) + \frac{1}{2}\left(-9.81\ \text{m/s}^2\right)(3.00\ \text{s})^2 \\
d_y &= +13.31\ \text{m} \\
\text{position} &= 0.0\ \text{m} + 13.31\ \text{m} = 13.31\ \text{m} \\
&= 13.3\ \text{m}
\end{aligned}
$$

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